Student Name: ______________________ Score:

Answer key

Balls in a container

	Work Space
There are 5 white balls, 8 red balls, 7 yellow balls and 4 green balls in a container. A ball is chosen at random. What is the probability of choosing red? Answer: $\frac{1}{3}$ What is the probability of choosing green? Answer: $\frac{1}{6}$ What is the probability of choosing either red or white? Answer: $\frac{13}{24}$ What is the probability of choosing neither white nor green? Answer: $\frac{5}{8}$ What is the probability of choosing a ball other than yellow? Answer: $\frac{17}{24}$ What is the probability of choosing black? Answer: 0	

PROBABILITY PROBLEMS SOLUTIONS

1 Probability can be recorded in words or using fractions, decimals or percentages.

a There is only **one** card showing a 6. P(the number 6) = 1 in 20 $= \frac{1}{20}$ $= 0.05$ $= 5\%$	**b** There are 6 multiples of 3: {3, 6, 9, 12, 15, 18} P(multiple of 3) = 6 in 20 or 3 in 10 $= \frac{6}{20}$ or $\frac{3}{10}$ $= 0.3$ $= 30\%$
c The prime numbers are: {2, 3, 5, 7, 11, 13, 17, 19} P(prime number) = 8 in 20 or 2 in 5 $= \frac{8}{20}$ or $\frac{2}{5}$ $= 0.4$ $= 40\%$	**d** This is the **complement** of selecting a prime. Use the probability of selecting a prime number. The probabilities add to 1. P(not prime) = 1 – P(prime) $= \frac{3}{5}$ $= 0.6$ $= 60\%$

2 The favourable outcomes are {3, 13, 23, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43}

P(at least one 3) = 14 out of 45

$= \frac{14}{45}$

$= 0.3\dot{1}$ (Note the repeater sign meaning 0.311111111111...)

$= 31\frac{1}{9}\%$ or $31.\dot{1}\%$

3 The three probabilities must add to 1.

$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$

$=\frac{5}{6}$

$1-\frac{5}{6}=\frac{1}{6}$

P(yellow) = $\frac{1}{6}$

4 If there is a 75% chance of selecting a red beetle then there is a 25% chance of selecting a blue beetle.

75% = 24 red beetles

25% = 8 blue beetles

100% = 32 beetles

There are 32 beetles altogether.

5 Arun's favourable outcomes are $\{1, 2, 3, 4, 5, 6, 7\}$.

Sally's favourable outcomes are $\{1, 2, 3, 4, 5\}$.

P(Arun winning) = $\frac{7}{9}$ P(Sally winning) = $\frac{5}{6}$

To compare the two fractions, you can convert them to decimals, percentages or fractions with common denominators.

Decimals	Percentages	Fractions
$\frac{7}{9} = 0.\dot{7}$ $\frac{5}{6} = 0.8\dot{3}$	$\frac{7}{9} = 77.\dot{7}\%$ $\frac{5}{6} = 83.\dot{3}\%$	$\frac{7}{9} = \frac{14}{18}$ $\frac{5}{6} = \frac{15}{18}$

Sally has the greater chance of winning.

6 One in five means there were originally 5 dark chocolates out of 25.

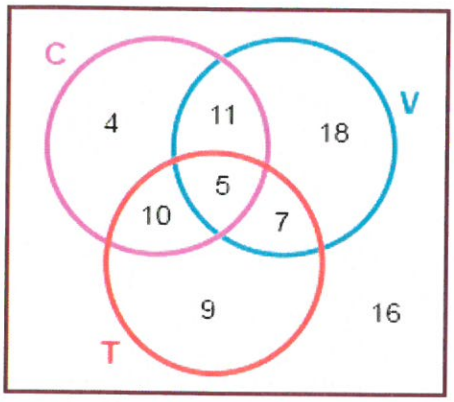
After one dark chocolate is eaten, there are 4 dark chocolates out of 24.

$$P(\text{dark}) = \frac{4}{24} \text{ or } \frac{1}{6}$$
$$= 0.1\dot{6}$$
$$= 16.\dot{6}\%$$

PROBABILITIES FROM DATA DISPLAYS

SOLUTIONS

TASK 1 Use a Venn diagram to find probabilities



There are 80 members in the travel club.

a P(did not visit any of the 3 countries) $= \frac{16}{80} = \frac{1}{5}$

b P(visited all 3 countries) $= \frac{5}{80} = \frac{1}{16}$

c P(visited China) $= \frac{30}{80} = \frac{3}{8}$

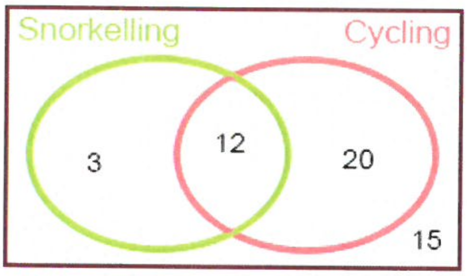
d P(only visited China) $= \frac{4}{80} = \frac{1}{20}$

e P(visited at least 2 of the countries) $= \frac{11+5+7+10}{80} = \frac{33}{80}$

f P(visited only one country) $= \frac{4+18+9}{80} = \frac{31}{80}$

g P(visited Vietnam and Thailand but not China) $= \frac{7}{80}$

TASK 2 Create a diagram or table to find probabilities



	Sport		
	Cycling	Not cycling	Totals
Snorkelling	12	3	15
Not snorkelling	20	15	35
Totals	32	18	50

a P(cycling but not snorkelling) $= \frac{20}{50} = \frac{2}{5}$

b P(only one of these sports) $= \frac{20+3}{50} = \frac{23}{50}$

c P(at least one of the sports) $= \frac{20+3+12}{50} = \frac{35}{50} = \frac{7}{10}$

d P(neither of the sports) $= \frac{15}{50} = \frac{3}{10}$

TASK 3 Use a two-way table to find probabilities

Janine's books

	Fiction	Non-fiction	Totals
Hardcover	13	44	57
Softcover	89	7	96
Totals	102	51	153

a $P(\text{fiction}) = \frac{102}{153} = \frac{2}{3}$

b $P(\text{hardcover}) = \frac{57}{153} = \frac{19}{51}$

c $P(\text{non-fiction}) = \frac{51}{153} = \frac{1}{3}$

d $P(\text{softcover}) = \frac{96}{153} = \frac{32}{51}$

e $P(\text{fiction and hardcover}) = \frac{13}{153}$

f $P(\text{non-fiction and softcover}) = \frac{7}{153}$

g $P(\text{fiction and softcover}) = \frac{89}{153}$

h $P(\text{neither fiction nor hardcover}) = \frac{7}{153}$

i $P(\text{either fiction or softcover}) = \frac{13+7+89}{153} = \frac{109}{153}$

Also, this is the complement of:

$P(\text{non-fiction and hardcover}) = \frac{44}{153}$

So you can use: $1 - \frac{44}{153} = \frac{109}{153}$

j $P(\text{either non-fiction or softcover}) = \frac{44+7+89}{153} = \frac{140}{153}$

Also, this is the complement of:

$P(\text{fiction and hardcover}) = \frac{13}{153}$

So you can use: $1 - \frac{13}{153} = \frac{140}{153}$

k There are 96 softcover books.

$P(\text{fiction}) = \frac{89}{96}$

l There are 102 fiction books.

$P(\text{softcover}) = \frac{89}{102}$

ADDITION RULE OF PROBABILITY **SOLUTIONS**

TASK 1 100 marbles and non-mutually exclusive (intersecting) sets

1 $100 \div 5 = 20$ and so there are 20 multiples of 5 in the jar.

$$\text{P(multiple of 5)} = \frac{20}{100}$$

2 $100 \div 8 = 12.5$ and so there are 12 multiples of 8 in the jar.

$$\text{P(multiple of 8)} = \frac{12}{100}$$

3 The first multiple of 5 and 8 is 40.

The next multiple of 5 and 8 is 80.

There are 2 multiples of 5 and 8 in the jar.

$$\text{P(multiple of 5 and 8)} = \frac{2}{100}$$

4 $\therefore$ P(multiple of 5 or 8) = P(multiple of 5) + P(multiple of 8) – P(multiple of 5 and 8)

$$= \frac{20}{100} + \frac{12}{100} - \frac{2}{100}$$

$$= \frac{30}{100}$$

$$= 0.3$$

TASK 2 100 marbles and mutually exclusive (non-intersecting) sets

There are 4 numbers between 85 and 90.

There are 10 square numbers in the jar. (The smallest is $1^2 = 1$ and the largest is $10^2 = 100$.)

P(number is between 85 and 90 or a square) = P(number between 85 and 90) + P(a square number)

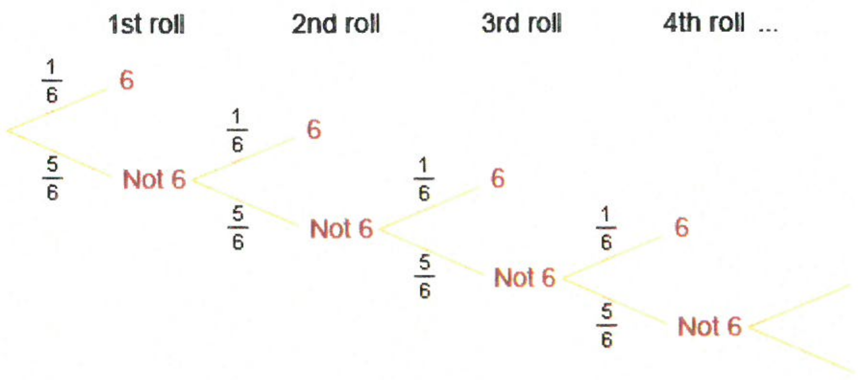
$$= \frac{4}{100} + \frac{10}{100}$$

$$= \frac{14}{100}$$

$$= 0.14$$

CHALLENGE **Roll a six**

Note: A probability tree showing this information is not symmetrical. Once Milu rolls a 6, she doesn't have another roll. So the tree branches out each step from the lower branch only (not 6).



1 $P(6) = \frac{1}{6}$

2 $P(\text{not } 6, 6) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

3 $P(\text{not } 6, \text{not } 6, 6) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

4 The pattern shows repeated factors of $\frac{5}{6}$ followed by one factor of $\frac{1}{6}$.

a (P not getting 6 until 10th roll) $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \ldots\ldots \times \frac{5}{6} \times \frac{1}{6}$ [There are **9** factors of $\frac{5}{6}$ here.]

$= (\frac{5}{6})^9 \times \frac{1}{6}$

b (P not getting 6 until 24th roll) $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \ldots\ldots \times \frac{5}{6} \times \frac{1}{6}$ [There are **24** factors of $\frac{5}{6}$ here.]

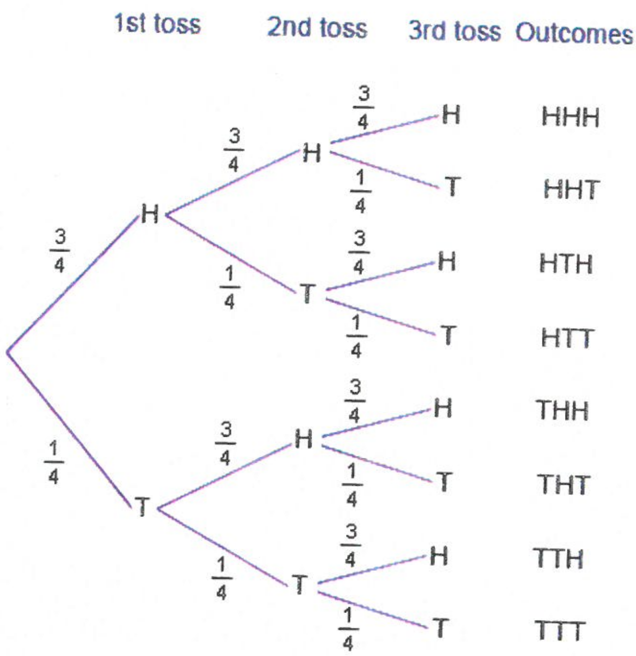
$= (\frac{5}{6})^{24} \times \frac{1}{6}$

TASK 2 **Coin flips**

1 **a** $P(H) = \frac{3}{4}$

b $P(T) = \frac{1}{4}$

2 See diagram.



3 **a** $P(HHH) = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

$= \frac{27}{64}$

b $P(TTT) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{64}$

c $P(HHT) = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

$= \frac{9}{64}$

d P(2 heads and 1 tail in any order) = P(HHT) + P(HTH) + P(THH)

$= (\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}) + (\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4})$

$= \frac{9}{64} + \frac{9}{64} + \frac{9}{64}$

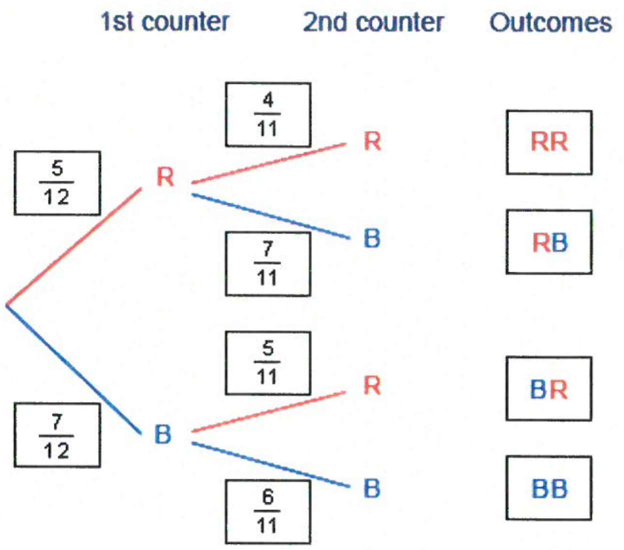
$= \frac{27}{64}$

PROBABILITY TREES WITHOUT REPLACEMENT SOLUTIONS

TASK 1 Counter counting

Since you do not replace the first counter in the bag before taking the second one, the numerators and denominators of the fraction probabilities will change from step 1 to step 2. This is called selection **without replacement**.

1



2 a $P(BB) = \frac{7}{12} \times \frac{6}{11}$

$= \frac{7}{22}$

b $P(\text{two counters same colour}) = P(RR) + P(BB)$

$= (\frac{5}{12} \times \frac{4}{11}) + \frac{7}{22}$

$= \frac{5}{33} + \frac{7}{22}$

$= \frac{31}{66}$

c $P(\text{different colours}) = P(RB) + P(BR)$

$= (\frac{5}{12} \times \frac{7}{11}) + (\frac{7}{12} \times \frac{5}{11})$

$= \frac{35}{132} + \frac{35}{132}$

$= \frac{35}{66}$

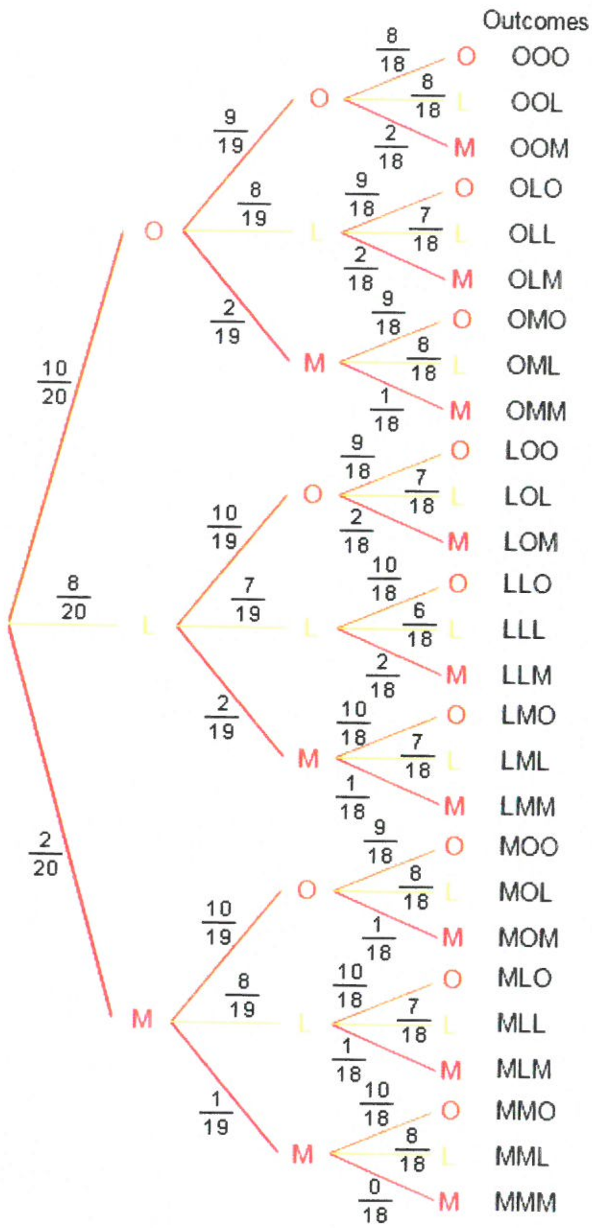
TASK 2 Flavour challenge

1 $n(O) = \frac{1}{2} \times 20 = 10$ $n(L) = \frac{2}{5} \times 20 = 8$ $n(M) = 20 - 10 - 8 = 2$

$\therefore P(M) = \frac{2}{20} = \frac{1}{10}$

2 See diagram.

Note: This question involves **dependent events**, so the fractional probabilities on the branches **change** from step to step.



3 $P(L \text{ then } M) = \frac{8}{20} \times \frac{2}{19}$

$= \frac{16}{380}$

$= \frac{4}{95}$

4 P(L and M, in any order)

$= P(LM) + P(ML)$

$= (\frac{8}{20} \times \frac{2}{19}) + (\frac{2}{20} \times \frac{8}{19})$

$= \frac{32}{380}$

$= \frac{8}{95}$

5 $P(MMM) = \frac{2}{20} \times \frac{1}{19} \times \frac{0}{18}$

$= 0$

Note: P(MMM) = 0 means that it is impossible to get 3 mandarin jubes—there are only 2 mandarin jubes in the packet.

6 P(all three jubes the same colour) $= P(OOO) + P(LLL) + P(MMM)$

$= (\frac{10}{20} \times \frac{9}{19} \times \frac{8}{18}) + (\frac{8}{20} \times \frac{7}{19} \times \frac{6}{18}) + 0$

$= \frac{720}{6840} + \frac{336}{6840}$

$= \frac{44}{285}$