# **General Mathematics**

2018 Subject Outline Stage 1



South Australian Board of SA Certificate of Education



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# INTRODUCTION

# SUBJECT DESCRIPTION

General Mathematics is a 10-credit subject or a 20-credit subject at Stage 1, and a 20-credit subject at Stage 2.

General Mathematics extends students' mathematical skills in ways that apply to practical problem-solving. A problem-based approach is integral to the development of mathematical models and the associated key ideas in the topics. These topics cover a diverse range of applications of mathematics, including personal financial management, measurement and trigonometry, the statistical investigation process, modelling using linear and non-linear functions, and discrete modelling using networks and matrices.

Successful completion of this subject at Stage 2 prepares students for entry to tertiary courses requiring a non-specialised background in mathematics.

# MATHEMATICAL OPTIONS

The diagram below represents the possible mathematical options that students might study at Stage 1 and Stage 2.



### Notes:

Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum per se is not a prerequisite for the study of Specialist Mathematics and Mathematical Methods. The essential aspects of 10A are included in the curriculum for Specialist Mathematics and Mathematical Methods.

Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

# CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:

- literacy
- numeracy
- information and communication technology (ICT) capability
- · critical and creative thinking
- personal and social capability
- · ethical understanding
- intercultural understanding.

## Literacy

In this subject students develop their literacy capability by, for example:

- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphical, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

## Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use skills, concepts, and technologies in a range of contexts that can be applied to:

- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- · using spatial sense and geometric reasoning
- · investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical and algebraic representations, and other mathematical models.

# Information and communication technology (ICT) capability

In this subject students develop their information and communication technology capability by, for example:

- understanding the role of electronic technology in the study of mathematics
- · making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant for study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice, and application; for example, to use data, address problems, and operate systems in particular situations.

# Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- · developing mathematical reasoning skills to think logically and make sense of the world
- · understanding how to make and test projections from mathematical models
- · interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

# Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- · meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

## **Ethical understanding**

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- · examining critically ways in which the media present particular perspectives
- · sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

## Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols which have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

# ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of highquality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

## SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 General Mathematics with a C grade or better, or 20 credits of Stage 2 General Mathematics with a C– grade or better, will meet the numeracy requirement of the SACE.

# LEARNING SCOPE AND REQUIREMENTS

## LEARNING REQUIREMENTS

The learning requirements describe the essential elements of Stage 1 General Mathematics. They summarise the knowledge, skills, and understandings that students are expected to develop and demonstrate through learning in the subject.

In this subject, students are expected to:

- 1. understand mathematical concepts and relationships
- 2. select and apply mathematical techniques and algorithms to analyse and solve problems, including forming and testing predictions
- 3. investigate and analyse mathematical information in a variety of contexts
- 4. interpret results, draw conclusions, and consider the reasonableness of solutions in context
- 5. make discerning use of electronic technology
- 6. communicate mathematically and present mathematical information in a variety of ways.

# CONTENT

Stage 1 General Mathematics is studied as a 10-credit or a 20-credit subject.

Students extend their mathematical skills in ways that apply to practical problem-solving and mathematical modelling in everyday contexts. A problem-based approach is integral to the development of mathematical skills and the associated key ideas in this subject.

Topics studied cover a range of applications of mathematics, including personal financial management, measurement and trigonometry, the statistical investigation process, modelling using linear functions, and discrete modelling using networks and matrices. In this subject, there is an emphasis on consolidating students' computational and algebraic skills and expanding their ability to reason and analyse mathematically.

Stage 1 General Mathematics consists of the following seven topics:

- Topic 1: Investing and borrowing
- Topic 2: Measurement
- Topic 3: Statistical investigation
- Topic 4: Applications of trigonometry
- Topic 5: Linear and exponential functions and their graphs
- Topic 6: Matrices and networks
- Topic 7: Open topic.

# Programming

Programs for a 10-credit subject must be made up of a selection of subtopics from at least three topics. Topics can be studied in their entirety or in part, taking into account student interests, and preparation for pathways into future study of mathematics.

Programs for a 20-credit subject must be made up of a selection of subtopics from at least six topics from the list.

The topics selected can be sequenced and structured to suit individual cohorts of students. The suggested order of the topics in the list is a guide only.

Topics 1 to 6 consist of a number of subtopics. These are presented in the subject outline in two columns as a series of key questions and key concepts, side by side with considerations for developing teaching and learning strategies.

Where a school chooses to undertake Topic 7: Open topic, the key questions and key concepts, considerations for developing teaching and learning strategies, and any subtopics will need to be developed.

The key questions and key concepts cover the content for teaching, learning, and assessment in this subject. The considerations for developing teaching and learning strategies are provided as a guide only.

A problem-based approach is integral to the development of the computational models and associated key concepts in each topic. Through key questions, students deepen their understanding of concepts and processes that relate to the mathematical models required to address the problems posed.

The considerations for developing teaching and learning strategies present problems and guidelines for sequencing the development of ideas. They also give an indication of the depth of treatment and emphases required.

# **Topic 1: Investing and borrowing**

Students discuss reasons for investing money and investigate using financial institutions and the share market as vehicles for investment of a sum of money. They calculate their expected returns from simple and compound interest investments using electronic technology (such as spreadsheets and financial packages in graphic calculators) and examine the effects of changing interest rates, terms, and investment balances. Students make comparisons between various scenarios and considerations of the limitations on the reliability of predictions made using simple and compound interest models.

Share market calculations include the costs of buying and returns from selling shares, break-even prices, and returns from dividends. Students make comparisons between the returns possible from share investments and those made in financial institutions. The effects of taxation and inflation on the return from a lump sum investment are investigated to determine whether real growth has occurred. Students consider the costs of borrowing money using credit or a personal loan, by accessing calculation tools on the internet.

## Subtopic 1.1: Investing for interest

## Key questions and key concepts

Why invest money in financial institutions? Where can money be invested?

- Discussion of financial institutions
- Fees and charges
- Types of investment

How is simple interest calculated, and in which situations is it used?

- · Using the simple interest formula to find the
  - simple interest
  - principal
  - interest rate
  - · time invested in years
  - total return

How does compound interest work?

How is compound interest calculated?

- Derivation of the compound interest formula
- Using the formula to find future value, interest earned, and present value

# Considerations for developing teaching and learning strategies

Students learn about the different types of financial institutions that can be used for investment (e.g. banks, credit unions, investment companies), the methods of investment they offer (e.g. term deposits, savings accounts), and the associated costs of investing in this way (i.e. fees and charges).

Discuss calculations involving rates and percentages as necessary.

Build spreadsheets to carry out simple interest calculations that could then be graphed, leading to a discussion of the linear relationship between time and amount of interest earned. Pose 'What if ...' questions to investigate the effects of changing the principal, interest rate, and time.

Students explore the formula for simple interest using examples of simple interest calculations, such as for term deposits.

Students recognise that simple interest is a percentage calculation multiplied by the number of years.

Using examples collected from financial institutions, students use a spreadsheet to examine the effect of compounding growth on an investment. They discuss the difference in the nature of this growth compared with simple interest.

Using technology, students examine the behaviour of the graph of time versus amount for compounding interest and explore how changes to interest rate and compounding periods affect the graph.

Guide students through the steps of the derivation of the formula for compound interest and then use it to further investigate the solutions to problems that involve finding future or present value or total interest earned on an investment.

## Key questions and key concepts

- Effects of changing the compounding period
- Annualised rates for comparison of investments
- · Using electronic technology to find the
  - future value
  - · present value
  - interest rate
  - + time
  - comparison rates on savings

Which is the better option: simple interest or compound interest?

# Considerations for developing teaching and learning strategies

When interest is compounded more often than annually, the compound interest formula can be adapted to take this into account. Students use this formula to explore the effects of changing the compounding period on the amount of interest earned. Another way to quantify the effect of more frequent compounding is to annualise the rate, making comparisons possible.

Technology provides an alternative way of solving problems. It can be used to find out how long it takes to save a certain amount, or the interest rate required. Provide a variety of problems for students to solve in practical contexts. They discuss the reasonableness of relying on such calculations because of the limitations of the model used (such as the assumption of a non-changing interest rate).

Present situations that allow students to compare the graphs of simple and compound interest growth for the same principal over time. Given a larger simple interest rate and a smaller compound interest rate, students use their graphs to determine when compound interest becomes the better option.

They also calculate the simple interest rate that, over a specified number of years, will earn the same amount as a given compound interest rate.

### Subtopic 1.2: Investing in shares

### Key questions and key concepts

How can the share market be used to make money from the money someone already has?

- · Share market information
- Costs and risks
- · Buying and selling shares
- Break-even price
- Using a brokerage rate

$$BE = \frac{b\left(1+1.1r\right)}{1-1.1r}$$

• Using a flat fee for brokerage

$$BE = \left(\frac{2.2f}{n}\right) + b$$

 Calculation of the dividend return from shares given the percentage dividend or the dividend per share

## Considerations for developing teaching and learning strategies

Students discuss the share market as an alternative to investing in a financial institution. This discussion includes where information on share investments can be found and the associated costs and risks involved.

Students calculate the cost of buying shares (including brokerage and the goods and services tax or GST) and the return from selling them at a later date. They also find the break-even price.

Dividends also provide income from share investments.

Using current information, students calculate the dividend income generated by a share portfolio.

## Subtopic 1.3: Return on investment

## Key questions and key concepts

- Expressing the return on an investment as a percentage of the original investment
- The effect of tax and inflation on real growth of an investment

# Considerations for developing teaching and learning strategies

To be able to compare one investment with another, students express the return as percentage growth.

To determine whether or not an investment has made real growth, consider, for example, the effect of tax and inflation. Introduce inflation as the equivalent of an annual compounding model and calculate it using either the formula or electronic technology.

## Subtopic 1.4: Costs of borrowing

## Key questions and key concepts

Why do many people use credit to buy items rather than saving for them?

What types of credit are available?

What is the total cost of using credit?

# Considerations for developing teaching and learning strategies

Students investigate forms of credit available, such as credit cards, store cards, line of credit, and discuss their advantages and disadvantages.

They discuss the extra costs to the purchaser who uses credit and calculate the total cost of using a credit card or a line of credit and compare with paying cash. Students investigate the cost of, for example, buying a TV or a computer on consumer credit.

Using internet-based bank loan calculators, students investigate the costs and time involved in repaying personal loans and research fees, charges, and hidden costs associated with loans.

Students investigate the effect of the inflation rate on the price of an item to find out how long it would take for the savings to equal the cost of the item. Expand this into a discussion on borrowing versus saving.

How much does a personal loan cost?

- Extra fees and charges
- Administration fees
- Interest

When is it better to borrow than save?

## **Topic 2: Measurement**

Students apply measurement techniques such as estimation, units of measurement, scientific notation, and measuring devices, and consider their accuracy. They extend their understanding of Pythagoras' theorem and use formulae to calculate the perimeter, area, and volume of standard plane and solid shapes, including triangles, quadrilaterals, circles, ellipses, prisms, pyramids, cylinders, cones, and spheres. This study is extended to compounds of these shapes. The estimation of irregular areas and volumes is considered by approximation using simple regular shapes or by applying Simpson's rule.

Students examine scales as they apply in practical contexts such as reading and making maps, plans, or models. Problems set in familiar contexts are used to develop students' understanding of the concept of rates as changes in related measurements, for example flow rates, density, or unit pricing.

## Subtopic 2.1: Application of measuring devices and units of measurement

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Application of common measuring devices, the metric system, its units and conversion between them.

How should accuracy be considered in measurement?

- Estimation and approximation
- Rounding off to a given number of significant figures
- Calculation of absolute and percentage errors using error tolerances

How are very large and very small values in measurement expressed?

Scientific notation

## Considerations for developing teaching and learning strategies

Students apply the most appropriate device and the associated metric units to measuring in a given situation. They consider the calculation of conversions between commonly used units.

Students estimate and measure quantities associated with familiar objects. The results are rounded to a sensible accuracy.

Students are aware of the accuracy or tolerances of measurements taken using various devices and use these to calculate either absolute or percentage (relative) errors of measurement. They discuss the implications of such errors in context.

Students correctly interpret both large and small measurements given in scientific notation on a calculator or spreadsheet.

## Subtopic 2.2: Perimeter and area of plane shapes

Key questions and key concepts	Considerations for develo and learning strat
How can we use Pythagoras' theorem to solve problems involving right-angled triangles?	Students will be familiar with Py theorem from previous years. T understanding of its application problems set in practical contex two- and three-dimensional sha
<ul> <li>How can knowing the perimeter and area of a two-dimensional shape help with solving a problem?</li> <li>Calculating circumferences and perimeters of standard and composite shapes (including circles, sectors, quadrilaterals, and triangles)</li> <li>Calculating areas of standard and composite shapes (including circles, sectors, quadrilaterals, ovals, trapeziums, and triangles)</li> </ul>	Students calculate perimeters a shapes. The focus is on solving problems set in familiar context complexity of the shapes involv
<ul> <li>Converting between units of measurement for area</li> </ul>	Students use appropriate units able to convert between square square metres, square kilometre
How can the area of an irregular plane shape be estimated?	Students practise approximating by superimposing a simple mat such as a circle, oval, rectangle

Approximation using a simple mathematical shape (circle, oval, rectangle, triangle, etc.)

## ping teaching egies

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and area of plane practical s, with increasing ed.

for area and are centimetres, es, and hectares.

g irregular areas hematical shape, e, or triangle, and balancing the 'overlaps' (area included and excluded) to enable them to calculate a reasonable estimate.

Compare the efficiency of this method with another, such as counting squares on an overlaid grid.

## Key questions and key concepts

Simpson's rule

$$A = \frac{1}{3}w(d_1 + 4d_2 + d_3)$$

## Considerations for developing teaching and learning strategies

Simpson's rule provides a useful formula for estimating the area of a shape with an irregular curved boundary.

If this rule is applied successively to an area divided into any even number of regions, the formula becomes:

Simpson's rule:

$$A = \frac{w}{3} \left( L_0 + 4L_1 + 2L_2 + \ldots + 4L_{n-1} + L_n \right)$$

where:

w = distance between offsets

 $L_k$  = length of the *k*th offset

To use this method accurately  $L_0$  and  $L_n$  must be the measurements of the offsets at the ends of the baseline, even if these lengths are of size zero. The offsets must divide the shape into an even number of regions of equal width. Cases where an end offset distance is zero must be dealt with correctly.

### Subtopic 2.3: Volume and surface area of solids

## Key questions and key concepts

How is the amount of space an object occupies or the amount of liquid a container will hold determined?

- Calculating volume or capacity for standard and composite solids including prisms, cylinders, cones, pyramids, and spheres.
- Converting between units for volume and capacity
- Estimating the volume of an irregular solid using an appropriate mathematical model

## Considerations for developing teaching and learning strategies

Students take measurements of prisms (e.g. a locker, dirt in a flower bed, the interior of a shed) or cylinders (e.g. a rainwater tank, a fire extinguisher) and calculate their volumes, using formulae and correct units of measurement. They extend their understanding to tapered shapes such as cones and pyramids, dealing with the sphere as a special case.

Skills and techniques are developed by solving practical problems in context. Students investigate the formulae for less common solids such as the frustum of a cone or pyramid, or the cap of a sphere.

Students use appropriate units for volume and capacity and convert between cubic centimetres, cubic metres, millilitres, litres, and kilolitres.

Irregular volumes are calculated using either Simpson's rule or approximation to a regular shape, to find an irregular base area that can be multiplied by an average height/depth (prismatic model) or one-third maximum height/depth (conical model).

## Key questions and key concepts

How is the area of the outside surface of a solid shape determined?

• Calculating surface area for standard and composite solids including prisms, cylinders, cones, pyramids, and spheres

# Considerations for developing teaching and learning strategies

For most of the standard solids, the surface area is the sum of the areas of the shapes that comprise its 'net'. This approach is used to arrive at surface area formulae for such solids. Students use such formulae when distinguishing between open and closed shapes (for instances an open pipe vs a solid cylinder). The sphere is treated as a special case.

## Subtopic 2.4: Scale and rates

### Key questions and key concepts

How does a scale factor work?

- Using a scale factor to calculate actual and scaled measurements
- Drawing scaled diagrams
- · Determining the scale factor needed or used
- · Scaling areas and volumes

What is a rate? What does it measure?

- Rates of change with time, particularly speed and flow rates
- · Other rates, particularly density
- Converting between units for a rate

## Considerations for developing teaching and learning strategies

Students work with maps and plans of different scale to calculate actual lengths or distances. They apply ratios and appropriate rounding off.

Students construct maps or plans, including those where they choose a scale themselves (e.g. a scale plan of the school for new students). A drawing software package could be used. Ground measurements could be used to determine the scale of a printout of a Google map of a local area.

Students obtain a scale factor and use it to solve scaling problems involving the calculation of areas of similar figures or surface areas and volumes of similar solids. An understanding of how scaling works when extended from simple linear measurement to two and three dimensions could be provided by considering the problems that can arise from testing scaled models in engineering or investigating the change in the cross-sectional area of the leg bones of animals as their weight increases.

Students investigate the idea of a rate as the change of one measurement with respect to another in practical situations. Speed and density are the most obvious of these; however, items/factors such as the 'gsm' rate for the thickness of paper or card, or flow rate from a tap, provide useful contexts for investigation.

Conversion between units for rates (e.g. m/s and km/h) is an important aspect of this subtopic.

# **Topic 3: Statistical investigation**

This topic begins with consideration of the structure of the process of statistical investigation from the collection of data using various methods of sampling. It proceeds to analysis using measures of central location and spread, to the formation of conjectures and the drawing of conclusions based on that analysis.

In sampling, there is emphasis on the importance of eliminating bias as well as ensuring the validity and reliability of results. Analysis of data incorporates its representation in tabular and graphical form (stem-and-leaf plots, box-and-whisker diagrams, and histograms) and the calculation of summary statistics from the sample.

Students learn to form conjectures that are supported or refuted by a logical argument, using justification from the results of their analysis. The suitability of the statistical tools and measures used in the solution of the problems is emphasised throughout this topic. Electronic technology is used to aid in the statistical investigation process.

## Subtopic 3.1: The statistical investigation process

## Key questions and key concepts

What are some examples of situations in which statistics are used to analyse and investigate problems?

The statistical process:

- · identifying the problem
- · formulating the method of investigation
- · collecting data
- analysing the data
- interpreting the results and forming a conjecture
- · considering the underlying assumptions

## Considerations for developing teaching and learning strategies

Students collect examples of data and statistics reported in the media.

Students consider the statistical process that underlies the production of the examples they have collected. They analyse individual items with a view to identifying the context, the problem being solved or investigated, the statistics used, and the data collected.

Students recognise the conjecture they are being asked to accept, and question the underlying assumptions that might have been made in the analysis. Useful contexts are provided by advertisements that use statistics.

Students discuss whether or not they are convinced by the information and arguments that are presented to them.

In the process of analysing items, students read and interpret data presented in a variety of ways.

# Subtopic 3.2: Sampling and collecting data

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is a sample and what is the purpose of sampling?	Students handle raw data with a clear purpose. This may be in the form of verifying a claim (e.g. 'On average the eggs we buy from the supermarket weigh 55 grams') or supporting a conjecture about the outcome of an experiment. The focus is on strengthening the statistical arguments used to support conjectures.
	Students develop the understanding that sampling is undertaken to reduce the expense in cost and/or time of assessing an entire population.
What is bias and how can it occur in sampling?	Students discuss the ways in which bias could affect the reliability of a sample yielding statistics that reflect those of the population.
<ul><li>What methods of sampling are there?</li><li>Simple random, stratified, and systematic sampling methods</li></ul>	Using appropriate sampling methods, students select samples of different sizes from a single large population for which the summary statistics are known. As the statistical investigation process is developed through the next three subtopics, discuss the effects of sample size.

## Subtopic 3.3: Classifying and organising data

## Key questions and key concepts

### Categorical data

- Ordinal
- Nominal

### Numerical data

- Discrete
- Continuous

How can data of the different types be appropriately organised and displayed?

- Categorical data tables and bar or pie charts
- Numerical data dot plot, stem plot, histogram

What is an outlier? How should outliers be dealt with?

## Considerations for developing teaching and learning strategies

Students become aware of the different measurement levels associated with categorical and numerical data and that there are appropriate ways to analyse and present these different levels. Specifically:

- Nominal or ordinal data can be summarised in a table of counts or proportions, from which a bar chart or pie chart can be drawn.
- Numerical data can be summarised in a frequency distribution table from which a dot plot, stem plot, or histogram can be drawn.

Students examine a range of types of data set, and use technology to present summaries of data whenever possible. They discuss the advantages and disadvantages of the various choices, and select the most appropriate form of presentation for a particular set of data. They discuss misleading representations.

Outliers would be identified visually in numerical data at this point (with a more formal definition to be given later, once measures of location and spread have been introduced). Decisions are made about how best to deal with them, remembering that only values that can be shown to be invalid should be removed from the data.

## Subtopic 3.4: The shape, location, and spread of distributions of numerical data

Key questions and key concepts	Considerations for developing teaching and learning strategies
What does the distribution of data within a data set look like?	Students identify the shapes of distributions of data, using stem plots, dot plots, or histograms. For single sets of data (e.g. the 'average egg' claim), students should look for the placement of the peak in the graph. For related sets of data (e.g. eggs from black vs white hens), they also look for differences in the characteristics of the shapes of the distributions — symmetry, skewedness, bimodality, and so on.
<ul><li>What is meant by 'average'?</li><li>Measures of central location (median and mean)</li></ul>	Students examine sets of data, considering the appropriateness of using median or mean (or possibly modal class) as a measure of average. They are reminded that measures of central location are valid for use only with data measured on an interval scale.
How do you decide on the most appropriate measure of 'average'? When can these measures become unreliable or misleading?	Discussion is supported by carefully chosen examples of what can distort the different measures of the centre of a distribution. They allow students to choose the one most appropriate for a given purpose and a given set of data. The effect of outliers on measures of centre is discussed.
Do sets of data with the same 'average' necessarily tell the same story? • Box-and-whisker plots	Students become aware that the centre, on its own, is of limited use as the descriptor of a distribution, but that it can be used to compare two sets of data or to compare a single set of data with a standard. The egg examples could be used here.
	Supported by carefully chosen examples and appropriate visual representations, students discuss the differences that can still exist when the 'average' is the same for two or more sets of data. Box plots are reviewed or introduced at

 $UB = Q_3 + 1.5 \times IQR.$ 

The various measures of spread are introduced and their limitations discussed, particularly with respect to the influence of outliers. Note that, although it is useful to show how the standard deviation is calculated by hand so that students understand how it is derived, they use electronic technology to find this value in any assessment task.

this point and the boundaries for outliers calculated using  $LB = Q_1 - 1.5 \times IQR$ ,

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• Measures of spread (range, interquartile

range, standard deviation)

• Outliers

## Key questions and key concepts

What influence does sample size have on the reliability of findings?

 Sample statistics compared with population parameters

# Considerations for developing teaching and learning strategies

Given some sample statistics, students can compare these values with known population parameters for different-sized samples to gain an understanding of the possible impact of sample size on the reliability of sample statistics as predictors of population parameters.

## Subtopic 3.5: Forming and supporting conjectures across two or more groups

### Key questions and key concepts

How do the statistical techniques and measures learnt help to argue whether a claim is true or false?

Analysis of numerical data:

- graphical representation
- · dealing with outliers
- shape of the distribution(s)
- · measures of centre and spread
- argument to support the conjecture

Analysis of categorical data:

- table of counts
- graphical representation
- identification of the mode
- · calculation of proportions
- · argument to support the conjecture

# Considerations for developing teaching and learning strategies

This subtopic draws all the threads of the statistical process together. Students investigate questions of interest using real data.

For numerical data, graphical tools for comparison include back-to-back stem plots, box plots with a common scale, or superimposed histograms.

For categorical data, the process is arguably simpler as the claim will be proportional (e.g. 'Most students use Brand X toothpaste' or 'Red cars are more popular than blue ones').

In arguing the truth of a conjecture, students consider the origin of the data and the sampling methods used (if they are known). Students understand that if the conjecture is supported by the sample this does not automatically make it true for the population from which the sample was taken.

# **Topic 4: Applications of trigonometry**

This topic focuses on the calculations involved in triangle geometry and their many applications in practical contexts such as construction, surveying, design, and navigation. An understanding of similarity and right triangle geometry leads students to the development of formulae for the calculation of the area of a triangle.

Non-right triangle trigonometry is introduced through the derivation of the cosine rule from

Pythagoras' theorem and the sine rule from the triangle area formula,  $A = \frac{1}{2}ab\sin\theta$ .

Students investigate problems that involve solving for unknown sides and angles in triangles found in both two- and three-dimensional situations. They consider cases where the data presented in sine rule problems may be ambiguous.

# Subtopic 4.1: Similarity

Key questions and key concepts	Considerations for developing teaching and learning strategies
In what kinds of problems are triangles important?	Plane figures form the basis of two-dimensional Euclidean geometry and triangles are the simplest of these. The importance of understanding the mathematics of triangles can be demonstrated by considering a variety of problems in contexts such as construction, design, surveying, and navigation.
How many measurements are required to determine a triangle uniquely?	Students are already familiar with the names and properties of different types of triangles. Through practical investigation they discover that, in most instances, three measurements involving side lengths and/or angles will determine a unique triangle. Discuss the exceptions to this (i.e. angle-angle-angle or AAA and side-side-angle or SSA) using specific examples.
Under what conditions can two triangles be proved to be similar?	Students understand that similar figures are in proportion to one another; that is, one is an enlargement of the other with a scale factor relating corresponding measurements.
	The conditions for similarity of triangles are discovered through modification of the rules found above.
	Constructions are done by hand and/or using an interactive geometry software package to demonstrate to students the (in)validity of establishing similarity using different sets of conditions.
How can similarity be used to solve problems?	Students solve problems set in practical contexts by establishing similarity, setting up a proportion, and solving for the unknown side.

## Subtopic 4.2: Right triangle geometry

## Key questions and key concepts

What mathematical tools are there for solving problems involving right-angled triangles?

- · Pythagoras' theorem
- Trigonometric ratios

# Considerations for developing teaching and learning strategies

Pythagoras' theorem is also covered in Topic 2.

Students' understanding of the trigonometric ratios for right triangles is consolidated. The idea of similarity is critical to understanding why each angle has its own unique values of sine, cosine, and tangent.

Problems are presented in two- and threedimensional contexts and with practical activities where appropriate, for example:

- finding the height of an object, using an inclinometer
- finding the angle of inclination of the sun
- determining whether or not a volleyball court is truly rectangular
- calculating the length of ladder needed to safely reach an otherwise inaccessible spot
- calculating the vertical angle of a cone, given its diameter and height.

### Subtopic 4.3: Area of triangles

### Key questions and key concepts

How is the area of a non-right triangle found if the perpendicular to a side cannot be measured easily or accurately?

$$Area = \frac{1}{2}ab\sin C$$

How can the area of a triangle be determined from its three sides?

Heron's rule

Area = 
$$\sqrt{s(s-a)(s-b)(s-c)}$$
  
where  $s = \frac{a+b+c}{2}$ 

## Considerations for developing teaching and learning strategies

Using right-angled triangles and trigonometry, students are led through the derivation of the

formula, 
$$Area = \frac{1}{2}ab\sin C$$
.

A practical comparison measuring and finding the area of one triangle using each of the three angles is a useful exercise to demonstrate the veracity of the formula. It forms the basis of a discussion of the significance of errors of measurement.

Heron's rule is applied to the side measurements of the triangle above to show it as an alternative way of calculating area. (The derivation of Heron's rule is not required in this course but could be shown to students where appropriate.)

A variety of practical and contextual problems are posed, requiring students to decide which rule to use and/or which measurements to take to find a specified area.

### Subtopic 4.4: Solving problems with non-right triangles

Key questions and key concepts

How are problems solved in which the triangles involved are not right-angled?

The cosine rule

 Solving for the third side when two sides and the included angle are known

$$a^2 = b^2 + c^2 - 2bc\cos A$$

 Solving for angles when the three sides are known

The sine rule

 Solving triangles where two sides and the non-included angle are known

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

• Solving triangles where two angles and one side are known

 Solving problems involving direction and bearings

## Considerations for developing teaching and learning strategies

The need for tools to deal with non-right triangles is emphasised by posing problems in contexts such as surveying, building, navigation, and design. Students are asked how they would find the answers to these problems, using the skills they have learnt. Discuss the validity and/or shortcomings of methods such as scale drawing and trial and error.

Derivation of the cosine rule is shown, using Pythagoras' theorem. Students recognise that the cosine rule is a generalised version of Pythagoras' theorem with a correction factor for angles that are larger or smaller than 90°. Students understand why the cosine of obtuse angles is negative. (Alternatively, calculation for a few such angles would demonstrate this fact.)

Students rearrange the formula into a form that can be used to find unknown angles.

Draw from real situations problems requiring the finding of an unknown side or angle using the cosine rule, and pose them in context.

Show the derivation of the sine rule from the area formula (or otherwise).

Confirmation of the sine rule by direct measurement is useful.

Students find the solution of contextual problems drawn from real situations for an unknown side or angle, using the sine rule.

Draw from real situations the problems posed for finding an unknown side or angle using the sine rule, and interpret the answers in context.

Present for discussion ambiguous cases where there are two possible solutions from the given information.

Discuss situations where, due to rounding or because contradictory data has been given in the question, the sine or cosine value calculated is greater than one and hence no angle can be found.

Students understand bearings and how to interpret them so that practical problems involving navigation and angles of elevation and depression can be solved by applying trigonometry.

# Topic 5: Linear and exponential functions and their graphs

This topic focuses on developing the process of mathematical modelling. It examines linear and exponential functions through a study of the various forms in which such relationships can be represented — contextual, numerical, graphical, and algebraic. Students identify the links that allow them to move between these representations and learn to use electronic technology to analyse and solve problems and make predictions. To deepen their understanding and improve their facility with these concepts, students experience applications of linear and exponential functions in a wide variety of contexts.

## Subtopic 5.1: Linear functions and graphs

## Key questions and key concepts

What is the nature of a linear relationship?

 Successive addition or subtraction of a constant value

How can problems that involve linear functions be represented?

Contextual description

In what other ways can such a problem be represented?

- Numerical table of values
- Graphical representation
  - · Slope and intercepts in context
  - Determining *x* or *y* value from a linear graph, given the other corresponding value

# Considerations for developing teaching and learning strategies

Mathematics is used to create models of situations we want to study. These models are useful for studying relationships, observing patterns, and making predictions. Linear functions are the simplest of these models. The functions used in this subtopic are drawn from a range of practical contexts (e.g. taxi fares, simple interest, water rates, timed telephone call charges) where there is a constant rate of change. Students examine these contexts to identify the variables involved and the linear nature of their relationship.

Students begin this study by considering examples such as the way a plumber, who has a callout fee of \$40 and a rate of \$25 per hour, calculates the charge for a job. How can we use this information to solve problems such as 'How much should be charged for a job taking 4¼ hours?' or 'If the charge is \$520, how long did the job take?'. Students provide estimated answers without requiring a formal approach. Posing similar problems in more complex contexts and with less simple parameters leads them to ask, 'Is this the most efficient way of solving these problems?'.

Students create a systematic table of values and identify the growth pattern in the plumber's charge values. The labels for the two columns in the table are used to introduce the idea of independent and dependent variables. By organising the data in this way, students answer the questions posed above more confidently. Discuss the usefulness and limitations of using the table for finding interpolated and extrapolated values.

The pairs of values in the table are plotted onto a graph. Students discuss the linearity of the graph and interpret terms such as 'slope' and 'axis intercepts' in context. Students use the graph to determine answers to the questions posed above and compare them with the other answers found. Discuss limitations to the accuracy and efficiency of finding solutions when using a graph to find interpolated and extrapolated values.

## Key questions and key concepts

- Algebraic formula
  - Developing a linear formula from a word description
  - Substitution and evaluation
  - Rearrangement of linear equations
  - · Solving linear equations

What are the links between the four ways of representing a linear relationship?

## Considerations for developing teaching and learning strategies

Students express the relationship between hours worked (*x*) and amount charged (*y*) as a verbal rule and are able to appreciate its translation into an algebraic equation — namely y = 25x + 40. By substituting for either variable and rearranging the equation (if necessary), the problems posed at the beginning of the topic, and any others, can be solved efficiently.

Students understand the links between the four representations of the problem being studied; namely:

- the two parameters (\$40 call-out fee, \$25 per hour rate) in the contextual description
- the initial value of *y* = \$40 (for *x* = 0) and the constant increment (+\$25) in the *y*-values in the table
- the *y*-intercept and slope of the graph
- the constant term and coefficient of *x* in the formula.

This allows translation between any two of these forms when solving problems.

Students are exposed to a wide variety of problems posed in different forms and contexts to strengthen their understanding of the representations of the linear model and the links between them. Once students have gained an understanding of the concepts, introduce electronic technology to aid the construction of tables and graphs.

## Subtopic 5.2: Exponential functions and graphs

### Key questions and key concepts

What is geometric growth or decay?

- Successive multiplication by a constant positive value
- Powers

How does this kind of growth or decay differ from that seen in linear relationships?

What are the different representations for an exponential function and how do we move between them?

- Features of the graph
- The algebraic formula  $y = a.b^x$

How can the model be used to solve problems in context?

Compound interest

## Considerations for developing teaching and learning strategies

Students examine examples of sequences, some drawn from practical contexts, which display change by the multiplication of a positive constant (both greater and less than one). Power notation may need to be reviewed.

Some examples are growth of a colony of bacteria, compound interest, and radioactive decay. Students use powers to calculate terms in such sequences.

From the numerical values in the sequences, students observe and discuss the difference in the nature of this growth or decay compared with that of a linear relationship. By graphing the terms, they see both the exponential nature of the growth and the asymptotic nature of the decay.

Using a calculator, students discover that noninteger values can also be used for exponents allowing for the development of the exponential function.

Using contextual examples of exponential growth and decay, students revisit the four ways of representing a mathematical model:

- contextual
- numerical
- graphical
- algebraic.

They note the specific features associated with the exponential function, namely the *y*-intercept, asymptotic behaviour, and shape of the graph, and the form of the algebraic formula as well as the links between them.

Students derive the formula for compound interest and investigate how changes to the initial conditions, such as the compounding period, affect the growth and the future value.

Problems requiring the finding of present and future values, time, and interest rate are solved with electronic technology using either a graph or an equation solver (the use of the 'time value of money' or TVM (financial) package is addressed in Stage 2). Algebraic techniques that use logarithms and *n*th roots are *not* required.

## Key questions and key concepts

- Other growth contexts (population growth, inflation, etc.)
- Decay contexts (radioactive decay depreciation, cooling etc.)
- Finding percentage growth or decay

# Considerations for developing teaching and learning strategies

Pose other problems involving both growth and decay in a variety of contexts. These problems begin with a situation in which the algebraic model is either given or can be deduced. Once the model has been determined, it is used to find values for one variable given the value of the other, using either a graph or an equation solver where necessary. The algebraic solution of exponential equations using logarithms is not required.

Students interpret the 'b' parameter of an

exponential model  $(y = a.b^x)$  as a percentage

gain or loss per time period, which can be interpreted in the context of the problem.

## **Topic 6: Matrices and networks**

This topic introduces students to discrete mathematics through the application of matrices and graph theory to solving problems in familiar contexts.

Costing and stock management allows the idea of matrices and their arithmetic to be learnt in a context where it can be logically developed. Electronic technology is used for efficiency in calculations involving matrix multiplication.

In the area of graph theory students encounter a range of optimisation problems that can be represented in the form of a network. Algorithms are developed that enable them to find the number of paths, shortest or longest path, and minimum connection or maximum flow in a network.

In both areas of this topic students examine problems set in a variety of contexts and discuss the appropriateness of the models and the usefulness of the solutions found.

Subtopic 6.1: Mati	ix arithmetic and	costing applications
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Key questions and key concepts	Considerations for developing teaching and learning strategies
What is a matrix?	Introduce matrices and their arithmetic using concrete examples. For instance, suppose that a business has three different outlets from which its stock is sold and that this stock comprises four different components. The amount of stock that has been delivered to each outlet in a particular month, for instance, can be efficiently stored, displayed, changed, or retrieved if it is kept in the form of a labelled rectangular array; namely, in a matrix.
<ul><li>How is information organised in a matrix?</li><li>Columns and rows in a matrix</li><li>Order (or dimensions) of a matrix</li></ul>	Explain the definition of the order of a matrix as the number of rows by the number of columns.
In what ways can costing and stock information in matrix form be manipulated?	Initially students carry out all matrix arithmetic by hand to develop a clear understanding of the processes.
<ul><li>Adding and subtracting matrices</li><li>Multiplication by a scalar</li></ul>	The need for two matrices to be of the same order for them to be added or subtracted is quite clear when based in a context such as stock control. Each corresponding position in the matrices represents something unique and the way to combine the numbers when adding the matrices is obvious. Similarly, multiplying any matrix by a number (scalar) is clear in this context.
<ul> <li>Matrix multiplication</li> <li>using a row or column matrix</li> </ul>	The concept of matrix multiplication is complex. By using an example such as 'Column matrix $A$ represents the number of two different items (say chops and sausages ordered by Mario for a barbecue) and matrix $B$ is a row matrix containing the cost of each item', the idea of multiplying pairs of numbers and summing the result to find the total cost is developed.
	By looking at the structure of the calculations with respect to the labels on rows and columns of the two matrices, it can be seen how the chop and sausage labels are matched up and then disappear in the final answer (cost of a chop $\times$ no. of chops for Mario + cost of a sausage $\times$ no. of sausages for Mario = total cost for Mario).

Key questions and key concepts	Considerations for developing teaching and learning strategies
<ul> <li>using matrices of higher order</li> </ul>	Once the process of multiplying individual rows and columns is understood, matrices of higher order can be multiplied. Use technology judiciously for matrix calculations once students are familiar with how they are carried out.
	Place emphasis on the acceptance of the convention of why the costs are given in a row matrix and not a column matrix in the case of the last example. Also, use examples where matrix <i>A</i> is transposed and the cost matrix is a column matrix.
<ul> <li>multiplying by a row or column matrix of 1s</li> </ul>	Multiplying a rectangular matrix $A$ on the left by a row matrix of 1s (of appropriate size) has the useful property of summing the elements in each column of matrix $A$ . Similarly, multiplying on the right by a column matrix of 1s sums the rows.
<ul> <li>Using electronic technology to do matrix arithmetic</li> </ul>	Having understood the processes of matrix arithmetic, it is appropriate for students to use electronic technology to carry out most calculations; however, they should be judicious in its use and perform calculations by hand where this would be more efficient.
How can matrices be used to solve problems in costing and inventory control?	Students investigate costing and inventory problems set in a variety of practical contexts that require the use of a range of matrix arithmetic operations.
How useful is the matrix model?	Students consider how the matrix model for costing and inventory control is more or less useful than other methods of calculation.

# Subtopic 6.2: Networks

Key questions and key concepts	Considerations for developing teaching and learning strategies
What are networks?	Students examine network diagrams drawn from a variety of contexts. These could include, for example, flow charts, precedence diagrams, maps, family trees, results of a sporting competition, and social relationships.
<ul><li>What information is given in a network diagram?</li><li>Reading information from a network diagram</li></ul>	By identifying the context, students interpret the information being presented in a weighted and directed network diagram (e.g. distance between nodes, time of travel, direction of travel, capacity of an arc, winning player, social influence) and answer specific questions about a situation.
<ul> <li>Deducing relationships</li> </ul>	Students realise that a network shows relationships and interconnections that are not always spatial. A study of a precedence network of jobs that make up a complex task is used as a case in point.
<ul> <li>Using appropriate terminology</li> </ul>	The correct terminology is taught where it is relevant to the problems studied (e.g. arcs, nodes, directed and undirected networks, trees, circuits) to enable consistent and concise communication in the discussion of networks and network problems.
How can networks be used to represent situations in which there is a problem to be solved?	Students are assisted to see how the information in problems can be represented in network form.
Connectivity networks	For example:
Flow networks	• We have to drive between two given places. Which is the best way to go? Why?
	• The local council is planning road upgrades because a lot of traffic passes through our area on its way to somewhere else. Which are the best roads to upgrade, and why?
	• Students have asked for drinking fountains to be installed in specific locations at the school. What is the best way to connect them all to the rainwater supply?
How many paths are there through a directed network? • With and without restrictions	By beginning with using trial and error to find the number of paths through a directed network, students appreciate the efficiency of using the algorithm. The problems are extended to include restrictions. For example, avoiding a node or an arc (e.g. because of an accident or a burst water main) or having to use a specified node or arc (e.g. because someone has to be picked up on the way).

## Key questions and key concepts

What is the shortest or longest path through a network?

• With and without restrictions

What is the cheapest way to connect up a set of points if there is more than one option available?

 Spanning trees – using 'greedy' and Prim's algorithms to find the minimum spanning tree in a connectivity network

What is the maximum flow that can be achieved through a network of conduits?

· Use of an algorithm to find maximum flow

## Considerations for developing teaching and learning strategies

Students consider weighted networks where each arc incurs a 'cost' or 'profit'. They explore the idea of an optimal or shortest path which uses the least cost or gains the most profit (with and without restrictions) and apply the algorithm for finding it. They interpret the meaning and understand the limitations of the answers gained using this kind of simplified mathematical model.

Explore the idea of a tree being a connected network with no circuits (i.e. no redundancy).

Students explore both scaled and unscaled representations of minimum spanning tree problems when seeking a solution. More than one algorithm is available and students consider which might be best in a given situation.

Extensions to these problems take practical considerations into account. For instance:

- What if a connection cannot be made in a straight line?
- What happens to the best solution if extra nodes are connected to the system later?
- Is the optimal solution practical if there are limitations on how far any node in the network can be from a source node?

The flow considered could be freight, people, water, telephone calls, internet connections, or traffic.

The algorithm using the exhaustion of paths is easier than the Dedekind 'cuts' method for all but the simplest networks. The cuts method is, however, useful when considering upgrades to a system of flow. Extensions of these problems would deal mainly with upgrading a system by either creating a new connection or improving an existing one.

## **Topic 7: Open topic**

Schools may choose to develop a topic that is relevant to their local context.

When developing an open topic, teachers should ensure that it:

- is introduced with an overview that provides a contextual framework, with an emphasis on application of the mathematics in the context
- includes an outline of the key questions and key concepts, with some consideration of the teaching and learning strategies that best relate to these questions and ideas
- is divided into subtopics, with key questions and key concepts, where appropriate
- enables students, together with the other topics for study, to develop the knowledge, skills, and understanding to meet the learning requirements of the subject
- emphasises the appropriate use of electronic technology in the teaching, learning, and assessment
- consists of content of a standard comparable to other topics outlined in the Stage 1 General Mathematics subject outline.

The open topic should relate to the needs, interests, and context of the particular group of students for whom the topic is developed.

The open topic should encourage a problem-based approach to mathematics, as this is integral to the development of the mathematical models and associated key concepts in each topic. Through the statement of key questions and key concepts, teachers can develop the concepts and processes that relate to the mathematical models required to address the problems posed. The teaching and learning strategies should give an indication of the depth of treatment and emphases required.

The topic at Stage 1 may be used to introduce and develop skills for an open topic that schools intend to teach at Stage 2.

# **ASSESSMENT SCOPE AND REQUIREMENTS**

Assessment at Stage 1 is school based.

## **EVIDENCE OF LEARNING**

The following assessment types enable students to demonstrate their learning in Stage 1 General Mathematics:

Assessment Type 1: Skills and Applications Tasks Assessment Type 2: Mathematical Investigation.

For a 10-credit subject, students should provide evidence of their learning through four assessments. Each assessment type should have a weighting of at least 20%.

Students undertake:

- at least two skills and applications tasks
- · at least one mathematical investigation.

For a 20-credit subject, students should provide evidence of their learning through eight assessments. Each assessment type should have a weighting of at least 20%.

Students undertake:

- · at least four skills and applications tasks
- at least two mathematical investigations.

## ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by teachers to:

- · clarify for students what they need to learn
- design opportunities for students to provide evidence of their learning at the highest level of achievement.

The assessment design criteria consist of specific features that:

- · students need to demonstrate in their evidence of learning
- teachers look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole, give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

## **Concepts and Techniques**

The specific features are as follows:

- CT1 Knowledge and understanding of concepts and relationships.
- CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.
- CT3 Application of mathematical models.
- CT4 Use of electronic technology to find solutions to mathematical problems.

## **Reasoning and Communication**

The specific features are as follows:

- RC1 Interpretation of mathematical results.
- RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.
- RC3 Use of appropriate mathematical notation, representations, and terminology.
- RC4 Communication of mathematical ideas and reasoning to develop logical arguments.
- RC5 Forming and testing of predictions.

\* In this subject the forming and testing of predictions (RC5) is not intended to include formal mathematical proof.

# SCHOOL ASSESSMENT

# Assessment Type 1: Skills and Applications Tasks

For a 10-credit subject, students complete at least two skills and applications tasks. For a 20-credit subject, students complete at least four skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher. Skills and applications tasks may provide opportunities to form and test predictions. Students must be given the opportunity to form and test predictions in at least one assessment type.

Students find solutions to mathematical problems that may:

- · be routine, analytical, and/or interpretative
- · be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine and some analytical and/or interpretative problems.

Students provide explanations and use correct mathematical notation, terminology, and representations throughout the task.

Electronic technology may aid and enhance the solution of problems. The use of electronic technology and notes in the skills and applications task assessments is at the discretion of the teacher.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- · concepts and techniques
- reasoning and communication.

## Assessment Type 2: Mathematical Investigation

For a 10 credit subject, students complete at least one mathematical investigation. For a 20 credit subject, students complete at least two mathematical investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by the teacher, or by a student or group of students. Teachers should give students clear advice and instructions on setting and solving the mathematical investigation, and support students' progress in arriving at a mathematical solution. Where students initiate the mathematical investigation, teachers should give detailed guidelines on developing an investigation based on a context, theme, or topic, and give clear direction about the appropriateness of each student's choice.

If an investigation is undertaken by a group, students explore the problem and gather data together to develop a model or solution individually. Each student must submit an individual report.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. statistical packages, spreadsheets, CAD, accounting packages) to enhance their investigation.

In a report, students form and test predictions, interpret and justify results, summarise, and draw conclusions. Students are required to give appropriate explanations and arguments. The mathematical investigation may provide an opportunity to form and test predictions.

A report on the mathematical investigation may take a variety of forms, but would usually include the following:

- · an outline of the problem to be explored
- the method used to find a solution
- · the application of the mathematics, including
  - generation or collection of relevant data and/or information, with a summary of the process of collection
  - mathematical calculations and results, using appropriate representations
  - discussion and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

Each investigation report, excluding bibliography and appendices if used, must be a maximum of eight A4 pages if written, or the equivalent in multimodal form. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as two A4 pages reduced to fit on one A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- · concepts and techniques
- reasoning and communication.

# **PERFORMANCE STANDARDS**

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers refer to in deciding, on the basis of the evidence provided, how well students have demonstrated their learning.

During the teaching and learning program the teacher gives students feedback on, and makes decisions about, the quality of their learning, with reference to the performance standards.

Students can also refer to the performance standards to identify the knowledge, skills, and understanding that they have demonstrated and those specific features that they still need to demonstrate to reach their highest possible level of achievement.

At the student's completion of study of a subject, the teacher makes a decision about the quality of the student's learning by:

- · referring to the performance standards
- · taking into account the weighting given to each assessment type
- assigning a subject grade between A and E.

# Performance Standards for Stage 1 General Mathematics

	Concepts and Techniques	Reasoning and Communication
Α	Comprehensive knowledge and understanding of concepts and relationships.	Comprehensive interpretation of mathematical results in the context of the problem.
	Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variaty of contexts.	Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.
	Successful development and application of mathematical models to find concise and accurate solutions	Proficient and accurate use of appropriate mathematical notation, representations, and terminology.
	Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.
		Formation and testing of appropriate predictions, using sound mathematical evidence.
В	Some depth of knowledge and understanding of concepts and relationships.	Mostly appropriate interpretation of mathematical results in the context of the problem.
	Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.	Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.
	Attempted development and successful application of mathematical models to find mostly accurate solutions.	Mostly accurate use of appropriate mathematical notation, representations, and terminology.
	Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to	Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.
	routine and some complex problems.	Formation and testing of mostly appropriate predictions, using some mathematical evidence.
С	Generally competent knowledge and understanding of concepts and relationships.	Generally appropriate interpretation of mathematical results in the context of the problem.
	Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in	Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.
	Application of mathematical models to find generally accurate solutions.	Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.
	Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.
		Formation of an appropriate prediction and some attempt to test it using mathematical evidence.

	Concepts and Techniques	Reasoning and Communication
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted formation of a prediction with limited attempt to test it using mathematical evidence.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to form or test a prediction.

# **ASSESSMENT INTEGRITY**

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement in the school assessment are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 1 are available on the SACE website (www.sace.sa.edu.au).

# SUPPORT MATERIALS

## SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

# ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).