## **General Mathematics**

2018 Subject Outline Stage 2



South Australian Board of SA Certificate of Education



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## CONTENTS

Introduction	1
Subject description	1
Mathematical options	2
Capabilities	
Aboriginal and Torres Strait Islander knowledge, cultures, and perspectives	6
SACE numeracy requirement	6
Learning scope and requirements	7
Learning requirements	7
Content	7
Assessment scope and requirements	30
Evidence of learning	30
Assessment design criteria	30
School assessment	31
External assessment	33
Performance standards	34
Assessment integrity	37
Support materials	
Subject-specific advice	38
Advice on ethical study and research	38

## INTRODUCTION

### SUBJECT DESCRIPTION

General Mathematics is a 10-credit subject or a 20-credit subject at Stage 1, and a 20-credit subject at Stage 2.

General Mathematics extends students' mathematical skills in ways that apply to practical problem-solving. A problem-based approach is integral to the development of mathematical models and the associated key concepts in the topics. These topics cover a diverse range of applications of mathematics, including personal financial management, the statistical investigation process, modelling using linear and non-linear functions, and discrete modelling using networks and matrices.

Successful completion of this subject at Stage 2 prepares students for entry to tertiary courses requiring a non-specialised background in mathematics.

## **MATHEMATICAL OPTIONS**

The diagram below represents the possible mathematical options that students might study at Stage 1 and Stage 2.



#### Notes:

Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum per se is not a prerequisite for the study of Specialist Mathematics and Mathematical Methods. The essential aspects of 10A are included in the curriculum for Specialist Mathematics and Mathematical Methods.

Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

## CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:

- literacy
- numeracy
- information and communication technology (ICT) capability
- critical and creative thinking
- personal and social capability
- · ethical understanding
- intercultural understanding.

#### Literacy

In this subject students develop their literacy capability by, for example:

- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- · analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphical, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

#### Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use skills, concepts, and technologies in a range of contexts that can be applied to:

- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- · using spatial sense and geometric reasoning
- investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical and algebraic representations, and other mathematical models.

## Information and communication technology (ICT) capability

In this subject students develop their information and communication technology (ICT) capability by, for example:

- understanding the role of electronic technology in the study of mathematics
- · making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant for study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice, and application; for example, to use data, address problems, and operate systems in particular situations.

## Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- · developing mathematical reasoning skills to think logically and make sense of the world
- · understanding how to make and test projections from mathematical models
- interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

### Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- · meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

#### **Ethical understanding**

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- · examining critically ways in which the media present particular perspectives
- · sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

#### Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols which have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

# ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of highquality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

## SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 General Mathematics with a C grade or better, or 20 credits of Stage 2 General Mathematics with a C- grade or better, will meet the numeracy requirement of the SACE.

## LEARNING SCOPE AND REQUIREMENTS

### LEARNING REQUIREMENTS

The learning requirements summarise the knowledge, skills, and understanding that students are expected to develop and demonstrate through learning in Stage 2 General Mathematics.

In this subject, students are expected to:

- 1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
- 2. investigate and analyse mathematical information in a variety of contexts
- 3. recognise and apply the mathematical techniques needed when analysing and finding a solution to a problem, including the forming and testing of predictions
- 4. interpret results, draw conclusions, and reflect on the reasonableness of solutions in context
- 5. make discerning use of electronic technology to solve problems
- communicate mathematically and present mathematical information in a variety of ways.

### CONTENT

Stage 2 General Mathematics is a 20-credit subject.

Stage 2 General Mathematics offers students the opportunity to develop a strong understanding of the process of mathematical modelling and its application to problem-solving in everyday workplace contexts.

A problem-based approach is integral to the development of both the models and the associated key concepts in the topics. These topics cover a range of mathematical applications, including linear functions, matrices, statistics, finance, and optimisation.

Stage 2 General Mathematics consists of the following six topics:

- Topic 1: Modelling with linear relationships
- Topic 2: Modelling with matrices
- Topic 3: Statistical models
- Topic 4: Financial models
- Topic 5: Discrete models
- Topic 6: Open topic.

Students study five topics from the list of six topics above. All students must study Topics 1, 3, 4, and 5.

For the fifth topic, schools may:

- follow the content for Topic 2: Modelling with matrices as outlined in this document, or
- choose to develop an open topic.

Topics 1 to 5 consist of a number of subtopics. These are presented in the subject outline in two columns as a series of key questions and key concepts, side-by-side with considerations for developing teaching and learning strategies.

Where a school chooses to undertake Topic 6: Open topic, the key questions and key concepts, considerations for developing teaching and learning strategies, and any subtopics will need to be developed.

The key questions and key concepts cover the prescribed content for teaching, learning, and assessment in this subject. The considerations for developing teaching and learning strategies are provided as a guide only.

A problem-based approach is integral to the development of the computational models and associated key concepts in each topic. Through key questions, teachers develop the key concepts and processes that relate to the mathematical models required to address the problems posed.

The considerations for developing teaching and learning strategies present suitable problems and guidelines for sequencing the development of ideas. They also give an indication of the depth of treatment and emphases required.

Although the external examination (see Assessment Type 3) will be based on the key questions and key concepts outlined in the three topics specified for examination, the considerations for developing teaching and learning strategies may provide useful contexts for examination questions. Topics 3, 4, and 5 are the basis for the external examination.

Stage 2 General Mathematics prepares students for entry to tertiary courses requiring a non-specialised background in mathematics.

## **Topic 1: Modelling with linear relationships**

Students review the concepts of continuous linear functions studied in Topic 5: Linear and exponential functions and their graphs in Stage 1 General Mathematics and extend their understanding through the solution of problems involving simultaneous linear equations. Linear programming is introduced as a major application of linear functions. The solution of problems involving the interaction of two variables is investigated in depth. At first, students solve problems set in realistic contexts with which they are familiar. They are encouraged to solve these problems by trial and error before being presented with the linear programming algorithm, so that they can appreciate the dynamic nature of the problems.

Problems are posed in everyday contexts. Examples of situations where these techniques are used are discussed with, or researched by, students.

Students investigate the effects on the optimal solution of changing the initial parameters in some problems. For instance, what effect will changing the objective function or the constraints have on cost, profit, or wastage? In situations where there are multiple optimal solutions, students discuss the merits of the different choices beyond the value of the objective function. Students explore situations involving both discrete and continuous variables and understand how to deal appropriately with an optimal solution that is not achievable in a context where only discrete values of the variables are acceptable.

Students use electronic technology to support the efficient solution of pairs of simultaneous linear equations and the investigation of linear programming scenarios.

#### Subtopic 1.1: Simultaneous linear equations

#### Key questions and key concepts

What are the different ways a linear function can be represented and what are the links between them?

- Contextual description
- Numerical sequence
- Graph
- Algebraic formula

When a problem has two independent variables, how much information is required to determine a unique solution?

How can contextual problems involving simultaneous linear equations be solved efficiently?

- Using electronic technology by:
  - graphing
  - · using the equation solver functionality
- Non-unique solutions

#### Considerations for developing teaching and learning strategies

From studying linear functions in Stage 1 General Mathematics, students will already be familiar with the four ways of representing relationships and the links between them. This key question is used to review these concepts.

Students realise that information given in a problem may not always produce a unique answer. For example, if told a cycle shop display containing bikes and trikes has 9 cycles, a student cannot determine how many of them are bicycles and how many are tricycles. They can, however, narrow it down to a limited set of possibilities.

It is only with a further piece of information (for instance that the display also has 23 wheels) that students can determine exactly how many bicycles and tricycles are in the shop display.

Students begin by trying to solve such a problem by trial and error, listing numerical possibilities for each constraint and finding a pair that match. However, by considering the graphical and algebraic representations of the linear relationships involved, more efficient methods of solution are demonstrated.

Electronic technology (either by graphing or using the equation solver functionality) is used to solve problems set in a variety of practical contexts once students understand the methods involved. When solving these problems, students rearrange the linear relationships into appropriate forms for entering data into the technology they are using.

So that they can deal with them appropriately, students are made aware that situations can arise in which there is a non-unique solution to a pair of simultaneous linear equations. This can result from insufficient information (two equivalent equations) or inconsistent information (parallel lines) given in the original problem.

#### Subtopic 1.2: Linear programming

#### Key questions and key concepts

How can linear functions be used to optimise a situation where we have control of two variables?

- Setting up the constraints and objective function
- Graphing the feasible region
- · Finding the optimal solution

## Considerations for developing teaching and learning strategies

Students consider what might be optimal (e.g. maximum profit, minimum cost, efficient use of materials) in a specific situation where two variables are involved. For example:

• The Year 12 students have some materials left over from the making of their school jumpers. They decide to use these materials to make two types of stuffed toy, which they will sell to raise money for their formal. What is the best way to use the materials?

This problem requires a combination of two things as its solution. By suggesting possible combinations and testing for their feasibility, students gain an appreciation of the dynamic nature of the problem. The quantity of one type of toy made will affect the materials available to make the other.

At this point a better way of organising the information is introduced. If different combinations of the two toy designs that will completely use up one of the materials are plotted on a grid, the straight-line relationship becomes apparent and its linear function is deduced. By testing points from either side of the line in the equation, the idea of graphing inequalities is developed and the feasible region is constructed.

When the possible solutions have been found, students use trial and error to seek the best solution for several different objective functions. They deduce that these solutions always occur on the boundary of the region. Simultaneous equations are used to find the corner points of the feasible region.

The solution process can be formalised as a set of steps which are carried out by hand as well as with the aid of electronic technology:

- Formulate and graph the constraints with appropriate labels
- · Formulate the objective function
- Identify the feasible region and calculate its vertices
- Evaluate the objective function at each vertex
- Compare the values to find the optimal solution.

#### Key questions and key concepts

· Considering wastage

How do we deal with an optimal solution that is not achievable because only discrete values are allowed?

What happens to the optimal solution if the original parameters change?

## Considerations for developing teaching and learning strategies

Students identify which of the originally available resources are used up in the optimal solution and calculate the 'wastage' or what is left of the rest.

In some contexts the variables can only take certain discrete values (e.g. whole numbers of garments or half tablets). The optimal solution given by the algorithm may need to be adjusted to be achievable. In such a case students test the discrete points that surround the optimal point to find the best solution that is possible, being careful to stay within the feasible region.

Students make changes to the objective function and/or constraint parameters to investigate the effects on the optimal solution and wastage.

## **Topic 2: Modelling with matrices**

This topic continues the development of discrete mathematics begun in Topic 6: Matrices and networks in Stage 1 General Mathematics. Students apply matrices to solve problems in practical contexts.

Two practical applications of matrices are studied: connectivity of networks and transition problems. The first application shows how matrices are used to examine the efficiency or reliability of a network system by considering the number of subsidiary paths it contains. In the second application, future trends are predicted in situations where things change state over time with known probabilities.

In both cases students examine problems set in a variety of contexts, and discuss the appropriateness of the models and the usefulness of the solutions found. Electronic technology is used extensively for calculations involving matrix multiplication and for investigating the effects of altering initial parameters on the outcomes.

### Subtopic 2.1: Application of matrices to network problems

Key questions and key concepts	Considerations for developing teaching and learning strategies
<ul><li>How can a matrix be used to show the connections in a network?</li><li>Connectivity matrices</li></ul>	Students convert from a directed network diagram to its representation as a connectivity matrix of zeros and ones (usually), and vice versa.
<ul><li>How do matrix operations help to find the number of indirect connections in a network?</li><li>Powers of matrices and multi-stage connections</li></ul>	By looking at the structure of matrix multiplication, it can be seen that squaring the connectivity matrix counts the number of directed two-step connections between any pair of vertices in the network.
	It can thus be induced that higher powers of the connectivity matrix similarly count the number of higher order multi-step paths.
<ul> <li>Limitations of using higher powers</li> </ul>	Although knowing how many <i>n</i> -step paths there are between a pair of vertices ( <i>A</i> and <i>B</i> , say) is of some use, the information is limited because without reference to the network itself, it is not known what these paths are, nor whether they are sensible (e.g. a 3-step path <i>ABAB</i> is not efficient when the 1-step path <i>AB</i> would suffice).
<ul><li>Of what use are weighted sums of the powers of connectivity matrices?</li><li>Measures of efficiency or redundancy</li><li>Prediction in dominance relationships</li></ul>	In a communications network, sums of the powers of the connectivity matrix can be used to examine how well the network is connected or to find the minimum number of steps required for there to be a path between every pair of nodes. They are used to examine the effects of adding or removing one or more arcs in the network.
	In a dominance situation (such as a round-robin sporting tournament) weighted sums are used to rank the power of the players and predict the likely outcome of unplayed matches.
<ul> <li>Reasonableness of weightings and limitations of the model</li> </ul>	Because of underlying assumptions, there are limitations to using matrix models in situations such as those described above. The elements of a connectivity matrix show whether or not connections exist but do not take into account any other qualitative or quantitative properties of those connections.

Different weightings in a weighted sum could give different results, so their significance needs to be considered in the context of the problem.

In a dominance situation, powers higher than 2 allow for cycles (non-zero elements on the main diagonal) that may not be appropriate and therefore need to be dealt with carefully.

Key questions and key concepts	Considerations for developing teaching and learning strategies
<ul> <li>What is a transition matrix and what are its properties?</li> <li>2×2 systems</li> </ul>	When consumers buy different brands of a certain product, how can a company determine what its market share is likely to be in the long term? Although there is an element of unpredictability about what an individual might do from one purchase to the next, matrix methods can be used to predict overall trends if something is known about the buying patterns of the whole population.
	Beginning with a simple $2 \times 2$ situation, students construct the matrix ( <i>T</i> ) of transition percentages (as decimals) and discuss the reason that the rows must add to one.
How can future trends be predicted?	It can be helpful to demonstrate with physical objects (such as counters) how the consumers move from one brand to another over time, given the transition proportions in $T$ .
	The number of consumers who are buying each brand at a particular time can be represented as a row matrix ( $R$ ). The matrix product $RT$ will then give the number of consumers expected to buy each brand next time they buy the product. By continuing to multiply by $T$ , trends can be predicted further into the future.
<ul><li>What happens in the long run in a transition model?</li><li>The steady state</li></ul>	Multiplication of $R$ by successive powers of $T$ will show a convergence to a 'steady state', where the market shares no longer change. It should be noted that although the proportions of the population buying each brand are no longer changing, there are still consumers switching between brands. Through investigation, students establish that the distribution of customers in $R$ has no effect on the final market shares in the steady state — it is the proportions in $T$ alone that determine the steady-state behaviour.
	Calculating high powers of the matrix $T$ where the rows become identical will indicate the proportions of the population who are buying each brand in the steady-state market.
What effect do changes to the initial conditions have on the steady state?	Students investigate what happens to the steady state if the probabilities in the initial transition matrix are changed. The motivation for this change is kept in context (e.g. one company mounts a strong advertising campaign, another receives adverse publicity, or the price of one brand changes significantly).

## Subtopic 2.2: Application of matrices to transition problems

#### Key questions and key concepts

• 3×3 or higher-order systems

What are the limitations of the transition matrix model?

## Considerations for developing teaching and learning strategies

Once students have established an understanding of the  $2 \times 2$  transition matrix model in the market shares context, the problems are expanded to matrices of order 3 or higher and to a variety of other contexts.

In the matrix model it is necessary to assume that the transition probabilities are fixed over a significant period of time to reach the steady state. It is also assumed that the same customers are involved every time (i.e. it is a closed system) and that they all make a purchase in every time period. Quite often at least one of these assumptions is unrealistic. It is important for students to realise that although the model is mathematical, it is not necessarily accurate.

*Note*: While  $2 \times 2$  matrices are used to introduce transition matrices, it is expected that students will handle problems involving  $3 \times 3$  or higher-order matrices in Assessment Type 1: Skills and Applications Tasks and Assessment Type 2: Mathematical Investigations.

## **Topic 3: Statistical models**

The linear and exponential growth behaviours studied in Topic 5: Linear and exponential functions and their graphs in Stage 1 General Mathematics are observed in bivariate data. By using electronic technology and statistical tools such as scatter plots and regression to analyse such data, students find algebraic models and use them for predictive purposes.

The normal distribution is an important mathematical model for making predictions in many social, industrial, and scientific contexts. Students investigate the characteristics and nature of the normal distribution through data simulation and graphical representation. They use this model to solve problems and make predictions in a range of contexts where data is expected to be approximately normally distributed.

Students use electronic technology to support both calculations and presentation of their work throughout this topic.

#### Subtopic 3.1: Bivariate statistics

#### Key questions and key concepts

How can bivariate data be modelled when the relationship appears linear but is not perfect?

- The statistical investigation process
- Independent and dependent variables
- Scatter plots
- Association
- Correlation coefficients

- The effects of outliers
- Causality

What is the linear relationship between two variables?

- Linear regression
  - y = ax + b
  - identification and interpretation of the slope and intercept of the graph of the equation y = ax + b

#### Considerations for developing teaching and learning strategies

Students examine example sets of paired data and discuss the steps involved in the process of statistical investigation.

For these data sets students identify the independent (explanatory) and dependent (response) variables from the context (or recognise when two variables may be co-dependent — for instance, marks on mathematics and English tests). They construct an appropriately scaled scatter plot and use it to describe the direction (positive/negative), form (linear, non-linear), and strength (strong/moderate/weak) of any association observed between the variables.

Students use electronic technology to calculate the values of Pearson's correlation coefficient (r) and the coefficient of determination ( $r^2$ ) and use them to assess the strength of a linear association in terms of the explanatory variable.

As a guide,  $r^2 \ge 0.7$  is sufficiently large to proceed with using a least squares regression line for prediction.

Outliers are identified visually from the scatter plot. Removing them from the data can strengthen the correlation and improve the way the line of best fit predicts; however, there needs to be careful consideration of whether or not it is appropriate to do this. Outlying data should only be removed with reasonable justification.

Students discuss whether a strong correlation is enough to imply that there is a causal link between the variables. The other possible explanations are coincidence or that both variables are changing in response to a third.

If the correlation is strong enough and the trend is linear, it is appropriate to find, using electronic technology, the equation of the least squares line of best fit (linear regression) y = ax + b.

Students recognise and interpret the values of the parameters 'a'/slope and 'b'/y-intercept in the context of the problem.

#### Key questions and key concepts

Residual plots

Exponential regression

•  $y = a.b^x$ 

## Considerations for developing teaching and learning strategies

A residual plot is used to decide if a straight line is the best model for the data. A pattern observed in the residual plot indicates that the trend in the data is, in fact, curved and a nonlinear model might fit the data better. Large residual values also indicate that the linear model may not be appropriate.

If the scatter plot and/or the residual plot indicate that a linear model may not be appropriate, students can use electronic technology to test whether or not an exponential model works for the data.

When fitting an exponential model to data, students follow the same protocols used with fitting a linear model and interpret the parameters of the equation in the context of the problem (i.e. '*a*' as the initial value and '*b*' as the proportional rate of change expressed as a percentage increase or decrease).

*Note:* Electronic technology will usually give the option to express the exponential equation in

either  $y = a.b^x$  or  $y = a.e^{kx}$  form. Students are not required to use the second form. If the technology they are using only gives the

 $y = a.e^{kx}$  form, students will need to be taught

how to calculate  $b = e^k$ .

Once the model for the relationship between the variables has been determined, values of the independent (explanatory) variable are used to predict values for the dependent (response) variable (and vice versa), either between the known data limits (interpolation) or outside the known data limits (extrapolation). The reliability of such predictions is discussed with reference to the correlation statistic and the residual analysis and, in the case of extrapolation, to the validity of assuming that the conjectured trend will continue beyond the data limits.

interpretation of the values of 'a' and 'b'

Interpolation and extrapolation, reliability, and interpretation of predicted results

#### Subtopic 3.2: The normal distribution

#### Key questions and key concepts

What is the normal distribution and how does it arise?

- Parameters  $\mu$  (mean) and  $\sigma$  (standard deviation)
- · Bell shape and symmetry about the mean

Why do so many observed sets of data appear normally distributed?

• Quantities that arise as the sum of a large number of independent random variables can be modelled as normal distributions

Why are normal distributions important?

- The variation in many quantities occurs in an approximately normal manner
- Normal distributions may be used to make predictions and answer questions that relate to such quantities

What are the characteristics of the normal distribution and how can they be used for prediction?

- 68:95:99.7% rule
- Calculation of area under the curve, looking at the position of one, two, and three standard deviations from the mean
- · Calculation of non-standard proportions

#### Considerations for developing teaching and learning strategies

Students experience a range of data sets that illustrate both normal and non-normal characteristics, leading to a discussion of the characteristic shape of the bell curve and its symmetry about the mean.

To investigate one explanation for why and where normal distributions occur, students experience the building of a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers. This is set in the context of simulating the production of something like brick pavers, where several factors in the production process introduce random variations into the final quantity being measured (e.g. length or weight). By building in the factors one at a time, students observe the emergence of a bell-shaped curve in the distribution of the measurements.

Discuss other contexts where the same underlying process might be expected to occur, with reference to data samples. Some possibilities are volume of soft drink in a can, height of humans of the same age and gender, weight of jelly beans, and lifetime of batteries.

Students discuss how knowing the approximate distribution of data in the contexts above could be useful in making decisions. For instance, does a short child fall within a 'healthy' range or should they be investigated further for growth problems? What is the likelihood that a can of soft drink will be under-filled on the assembly line?

A refinement of the spreadsheet mentioned above allows students to see the features of normal distributions unfold.

Students understand the '68:95:99.7%' rule and use it to make predictions of approximate proportions or probabilities in context, given the mean and standard deviation. These calculations are done both with and without the use of electronic technology.

Using electronic technology, students find other proportions or probabilities that relate to non-integral multiples of the standard deviation from the mean.

### Key questions and key concepts

• Calculation of values on the distribution, given the area under the curve

# Considerations for developing teaching and learning strategies

Students reverse the process to find values on the distribution that bound a given area under the curve ('inverse normal' problems).

### **Topic 4: Financial models**

In this topic the focus is on the annuity model and its applications to investing and borrowing money. The broad areas of consideration are:

- · saving money for a future need by making regular deposits
- repayment of a reducing balance loan
- receiving an income stream from a lump-sum investment.

Students investigate the different types of saving plans, such as superannuation and long-term deposits. They consider the effects of bank and government charges, taxation, and inflation on savings plans. Students consider the costs of borrowing money, taking into account a range of variables (e.g. repayments, interest rates, term of the loan, compounding interval). They discuss mortgages, personal loans, pension annuities, and interest-only loans with sinking funds.

Students use the annuity model to investigate strategies for minimising interest paid on a loan or maximising the interest earned on an investment. The nature of the calculations involved requires use of electronic technology (via the graphic calculator financial package or spreadsheets) to aid efficient investigation.

#### Subtopic 4.1: Models for saving

#### Key questions and key concepts

How is the compound interest model used to plan for the future?

• Finding FV, PV, n, and I

How can the compound interest model be improved to make it more realistic and flexible?

What mathematics is used in calculating futurevalue annuities?

- Future value
- The regular deposit
- The number of periods
- The interest rate
- The value of the accumulating savings after a given period
- Total interest earned

What factors should be considered when selecting an investment?

- Interest as part of taxable income, including calculations
- The effects of inflation, including calculations
- · Institution and government charges

How can different investment structures be compared?

 Comparison of two or more investments involving nominal and/or flat interest by conversion to an equivalent annualised rate (effective rate)

How can a regular income be provided from savings?

- Annuities
- Superannuation

#### Considerations for developing teaching and learning strategies

Students use electronic technology to solve problems involving compound interest.

The concept of annuities is explored using electronic technology to solve more complex problems involving regular savings annuities.

Students investigate 'What if ...' questions with varying future values, payments, rates, and times. They discuss the limitations to the reasonableness of their results given the underlying assumptions made in the model.

Contexts include superannuation and long-term deposits.

The return from any investment is subject to many factors that can combine to erode the overall return. Students become aware of the impact of these factors when managing investments.

Students consider the effect of inflation on long-term investments.

Investment structures can differ in the type of interest paid, fee structure, or their rate of compounding the rates, and are not always immediately comparable. Effective annualised compounding rates allow for a common basis on which to make such comparisons.

Students consider the importance of effective rates in relation to advertisements for saving schemes.

Students consider how, by reversing the annuity savings model, a lump-sum deposit can be used to provide a regular income.

A variety of 'What if ...' scenarios are investigated and the limitations of the model discussed.

*Note*: In Assessment Type 3: Examination, the number of compounding periods per year will be equivalent to the number of payments per year for calculations of future value annuities and present value annuities. However, where technology permits, different payment and compounding periods can be investigated in class and considered for Assessment Type 2: Mathematical Investigations.

#### Subtopic 4.2: Models for borrowing

#### Key questions and key concepts

If money must be borrowed, how much will it cost?

## Considerations for developing teaching and learning strategies

Most students, at some stage in their lives, will borrow a significant sum of money. They consider how much this could cost in the long run to determine whether it would be better to save the money and pay cash, or borrow and repay the debt.

Loans are structured in different ways, often according to their purpose. Students obtain information from lending institutions about the types of personal, business, and home loans available.

Interest-only loans and sinking funds may be used by property investors or businesses. Students carry out the calculations associated with taking out an interest-only loan, and using a sinking fund to repay the principal. They discuss the possible advantages or disadvantages of using such a model.

Students use electronic technology to calculate various aspects of a loan. This process often highlights the real costs of a loan, particularly in its early stages.

Many different factors that affect the cost of a loan are under the control of the borrower. Students investigate the effects that changing one or more of these factors may have on the total interest paid.

Discussion includes the reasonableness and/or affordability of the various interest-minimisation strategies.

*Note*: In Assessment Type 3: Examination, the number of compounding periods per year will be equivalent to the number of payments per year for loans calculations. However, where technology permits, different payment and compounding periods can be investigated in class and considered for Assessment Type 2: Mathematical Investigations.

- Interest-only loans and sinking funds
- Reducing-balance loans
  - Finding the repayment for a given loan
  - · Calculating total interest paid
  - The size of an outstanding debt after a given time

How could the amount of interest paid on a loan be reduced?

Finding the effect of:

- · increasing the frequency of payments
- · increasing the value of the payments
- reducing the term of the loan
- paying a lump sum off the principal owing
- · changing interest rates
- using offset accounts

#### Key questions and key concepts

Is the nominal rate of interest quoted by a bank what is really being paid on a loan?

- Loan interest rates, including variable rate, fixed rate, and others
- Interest paid
- Calculation of the comparison rates for two or more loans to determine the most appropriate option

## Considerations for developing teaching and learning strategies

When they compare loan options, students investigate comparison rates that take into account the fees and charges connected with the loan, as well as the compounding period. Discussion focuses on the importance of factors other than the interest rate when choosing a loan.

*Note*: When calculating a comparison rate for a loan that attracts fees, the one-off set-up fee should be added to the initial present value, and the ongoing regular charges should be added to the payment figure.

### **Topic 5: Discrete models**

The focus of this topic is on network applications to the solution of problems involving critical path analysis and assignment problems. In critical path analysis, students determine the shortest time in which a complex task can be completed and identify the critical components of that task. To demonstrate the diversity of discrete models, students investigate assignment problems and learn the application of the Hungarian algorithm to their solution.

In both subtopics, before being presented with solution algorithms, students attempt to solve problems set in familiar contexts by trial and error so that they can appreciate the nature and complexity of those problems. Once the algorithms have been introduced, new problems are posed in broader contexts so that students understand that, although the calculations are relatively simple, the methods they are learning underpin some powerful techniques.

As part of their study, students investigate the effects of changing the initial conditions or parameters of problems with a view to improving the solutions. For instance, which jobs could be shortened to improve the minimum completion time?

The arithmetic computations required for the solution of the problems presented in this topic can be conducted without electronic technology.

#### Subtopic 5.1: Critical path analysis

#### Key questions and key concepts

If a job requires the completion of a series of tasks with set precedence, what is the minimum time in which this job can be finished?

- Precedence tables
- · Drawing directed networks
- Dummy links

For which of the tasks is it critical that there is no delay?

- · Forward and backward scan
- Minimum completion time
- Critical path
- Earliest and latest starting times for individual tasks
- Slack time

## Considerations for developing teaching and learning strategies

Students become acquainted with the idea of precedence in the flow of jobs that make up a complex task. Through a practical example, they explore how to create a precedence table that indicates which other jobs must be completed before a given job can start.

From a precedence table, a directed network can be drawn to represent the task. For straightforward networks this can be done by trial and error; however, students may benefit from being taught how to use a bipartite graph to work out the order in which to construct the nodes.

It is sometimes necessary to use dummy links in the network to show a given precedence correctly (e.g. when job E requires both A and Bto be complete but job C requires only B to be complete). Students gain an understanding that two different-looking networks may be topologically identical.

Once a network representation is available for a problem, students can determine the minimum completion time and critical jobs by finding the longest path through the network. They discuss the amount of leeway available in the starting time for a given job in the network, and what happens if time for a specific job is shortened or lengthened. They look for ways of reducing the minimum completion time in the context of a specific problem. Students discuss the reasonableness of their results and any limitations to the model in the context of the problem.

#### Subtopic 5.2: Assignment problems

#### Key questions and Key concepts

What are assignment problems?

How can assignments be arranged to give the optimum result?

- The Hungarian algorithm
  - Finding minimum cost

- · Finding maximum profit
- Non-square arrays

## Considerations for developing teaching and learning strategies

Assignment problems deal with allocating tasks in a way that minimises 'costs' (note that 'costs' can be measurements such as time or distance, as well as money). For example, if the times in which four swimmers each do 50 metres of each of the four different strokes are known, how should they be placed in a medley relay to minimise the total time for them to complete the race?

The Hungarian algorithm consists of a set of iterative steps to find the optimum solution of an assignment problem.

- Reduce the array of 'costs' in rows and then columns by subtraction of the minimum value.
- 2. Cover the zero elements with the minimum number of straight lines. If the number of lines used is the same as the order of the array, go to Step 4.
- Let *m* be the minimum uncovered element. The array is further reduced by subtracting *m* from all uncovered elements and adding *m* to any element covered by two lines. Return to Step 2.
- 4. There is an optimum assignment using only zeros in the augmented array. Apply this pattern to the original array. Note that there may be more than one optimal solution.

(See https://www.youtube.com/watch?v=dQDZ NHwuuOY (farnboroughmaths, 2013) for a YouTube clip demonstrating the Hungarian algorithm.)

The algorithm can be adapted to finding the assignment that gains the maximum 'profit' by effectively minimising profit lost.

In some problems the array is rectangular rather than square, for example when there are six possible contenders for the four positions in the medley relay swim team. In such cases the array is 'squared up' by adding dummy rows or columns before applying the algorithm.

## **Topic 6: Open topic**

Schools may choose to develop a topic that is relevant to their own local context. When this option is undertaken, the open topic developed replaces Topic 2: Modelling with matrices.

When developing an open topic, teachers should ensure that it:

- is introduced with an overview that provides a contextual framework, with an emphasis on application of the mathematics in the context
- includes an outline of the key questions and key concepts, with some consideration of the teaching and learning strategies that best relate to these questions and ideas
- is divided into subtopics, with key questions and key concepts, where appropriate
- enables students to develop the knowledge, skills, and understanding to meet the learning requirements of the subject, together with the other topics for study
- emphasises the appropriate use of electronic technology in teaching, learning, and assessment
- consists of content of an standard comparable to that of other topics outlined in the Stage 2 General Mathematics subject outline.

Topic 6: Open topic should relate to the needs and interests of the particular group of students for whom the topic is developed.

The topic should encourage a problem-based approach to mathematics as this is integral to the development of the mathematical models and associated key concepts in each topic. Through the statement of key questions and key concepts, teachers can develop the concepts and processes that relate to the mathematical models required to address the problems posed. The teaching and learning strategies should give an indication of the depth of treatment and emphases required.

## **ASSESSMENT SCOPE AND REQUIREMENTS**

All Stage 2 subjects have a school assessment component and an external assessment component.

## **EVIDENCE OF LEARNING**

The following assessment types enable students to demonstrate their learning in Stage 2 General Mathematics:

School assessment (70%)

- Assessment Type 1: Skills and Applications Tasks (40%)
- Assessment Type 2: Mathematical Investigations (30%)

#### External assessment (30%)

Assessment Type 3: Examination (30%)

Students provide evidence of their learning through eight assessments, including the external assessment component. Students undertake:

- five skills and applications tasks
- two mathematical investigations
- one examination.

### ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by:

- teachers to clarify for students what they need to learn
- teachers and assessors to design opportunities for students to provide evidence of their learning at the highest possible level of achievement.

The assessment design criteria consist of specific features that:

- · students should demonstrate in their learning
- teachers and assessors look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole, gives students opportunities to demonstrate each of the specific features by the completion of study of the subject.

### **Concepts and Techniques**

The specific features are as follows:

- CT1 Knowledge and understanding of concepts and relationships.
- CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.
- CT3 Application of mathematical models.
- CT4 Use of electronic technology to find solutions to mathematical problems.

### **Reasoning and Communication**

The specific features are as follows:

- RC1 Interpretation of mathematical results.
- RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.
- RC3 Use of appropriate mathematical notation, representations, and terminology.
- RC4 Communication of mathematical ideas and reasoning to develop logical arguments.
- RC5 Forming and testing of predictions.\*

\* In this subject the forming and testing of predictions (RC5) is not intended to include formal mathematical proof.

## SCHOOL ASSESSMENT

### Assessment Type 1: Skills and Applications Tasks (40%)

Students undertake five skills and applications tasks, including at least one skills and applications task from each of the non-examined topics.

Skills and applications tasks are completed under the direct supervision of a teacher.

The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

In the remaining skills and applications tasks, electronic technology and up to one A4 sheet of paper of handwritten notes (on one side only) may be used at the discretion of the teacher.

Students find solutions to mathematical questions that may:

- be routine, analytical, and/or interpretative
- · be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require the student to demonstrate an understanding of relevant mathematical ideas, facts, and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information and apply them to find solutions to routine, analytical, and/or interpretative problems. Some of these problems should be set in contexts, for example: social, scientific, economic, or historical.

Students provide explanations and arguments, and use mathematical notation, terminology, and representations correctly throughout the task.

Skills and applications tasks may provide opportunities to form and test predictions. Students must be given the opportunity to form and test predictions in at least one assessment task in the school assessment component.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- · concepts and techniques
- reasoning and communication.

### Assessment Type 2: Mathematical Investigations (30%)

Students complete two investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by the teacher, or by a student or group of students. Teachers should give students clear advice and instructions on setting and solving the mathematical investigation, and support students' progress in arriving at a mathematical solution. Where students initiate the mathematical investigation, teachers should give detailed guidelines for developing an investigation based on a context, theme, or topic, and give clear direction about the appropriateness of each student's choice.

If a mathematical investigation is undertaken by a group, students explore the problem and gather data together to develop a model or solution individually. Each student must submit an individual report.

Teachers may need to provide support and clear directions for the first investigation. However, the second investigation must be less directed and set within more open-ended contexts.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use mathematical and other software (e.g. statistical packages, spreadsheets, Computer Algebra Systems (CAS), accounting packages) to enhance their investigation. The generation of data and the exploration of patterns or the changing of parameters may provide an important focus. Notation, terminology, forms of representation of information gathered or produced, calculations, and results are important considerations.

In the report, students interpret and justify results, and draw conclusions. They support this process by giving appropriate explanations and arguments. A mathematical investigation may provide opportunities to form and test predictions. The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results using appropriate representations
  - discussion and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

Each investigation report, excluding bibliography and appendices if used, must be a maximum of 12 A4 pages if written, or the equivalent in multimodal form. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as two A4 pages reduced to fit on one A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- · concepts and techniques
- reasoning and communication.

## **EXTERNAL ASSESSMENT**

#### Assessment Type 3: Examination (30%)

Students undertake a 2-hour external examination in which they answer questions on the following three topics:

- Topic 3: Statistical models
- Topic 4: Financial models
- Topic 5: Discrete models.

The examination is based on the key questions and key concepts in Topics 3, 4, and 5. The considerations for developing teaching and learning strategies are provided as a guide only, although applications described under this heading may provide contexts for examination questions.

The examination consists of a range of problems, some focusing on knowledge, routine skills, and applications, and others focusing on analysis and interpretation. Students provide explanations and arguments, and use correct mathematical notation, terminology, and representations throughout the examination.

Students may take one unfolded A4 sheet (two sides) of handwritten notes into the examination room.

Students may have access to approved electronic technology during the external examination. However, students need to be discerning in their use of electronic technology to find solutions to questions/problems in examinations.

All specific features of the assessment design criteria for this subject may be assessed in the external examination.

### PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers and assessors refer to in deciding how well students have demonstrated their learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of each school assessment type, the teacher makes a decision about the quality of the student's learning by:

- · referring to the performance standards
- assigning a grade between A+ and E- for the assessment type.

The student's school assessment and external assessment are combined for a final result, which is reported as a grade between  $A_{+}$  and  $E_{-}$ .

## Performance Standards for Stage 2 General Mathematics

	Concepts and Techniques	Reasoning and Communication
Α	Comprehensive knowledge and understanding of concepts and relationships.	Comprehensive interpretation of mathematical results in the context of the problem.
	<ul> <li>Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.</li> <li>Successful development and application of mathematical models to find concise and accurate solutions.</li> <li>Appropriate and effective use of electronic technology to find accurate solutions to routine</li> </ul>	Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate
		mathematical notation, representations, and terminology.
		Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.
	and complex problems.	Formation and testing of appropriate predictions, using sound mathematical evidence.
в	Some depth of knowledge and understanding of concepts and relationships.	Mostly appropriate interpretation of mathematical results in the context of the problem.
	Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.	Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.
	Attempted development and successful application of mathematical models to find mostly accurate solutions.	Mostly accurate use of appropriate mathematical notation, representations, and terminology.
	Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.
		Formation and testing of mostly appropriate predictions, using some mathematical evidence.
С	Generally competent knowledge and understanding of concepts and relationships.	Generally appropriate interpretation of mathematical results in the context of the problem.
	Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in different contexts	Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.
	Application of mathematical models to find generally accurate solutions.	Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.
	Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.
		Formation of an appropriate prediction and some attempt to test it using mathematical evidence.

	Concepts and Techniques	Reasoning and Communication
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted formation of a prediction with limited attempt to test it using mathematical evidence.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to form or test a prediction.

## **ASSESSMENT INTEGRITY**

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement, in both the school assessment and the external assessment, are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 2 are available on the SACE website (www.sace.sa.edu.au)

## SUPPORT MATERIALS

### SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

## ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).