Specialist Mathematics

2018 Subject Outline

Stage 2

For teaching

- in Australian and SACE International schools from January 2018 to December 2018
- in SACE International schools only, from May/June 2018 to March 2019

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INTRODUCTION

SUBJECT DESCRIPTION

Specialist Mathematics is a 20-credit subject at Stage 2.

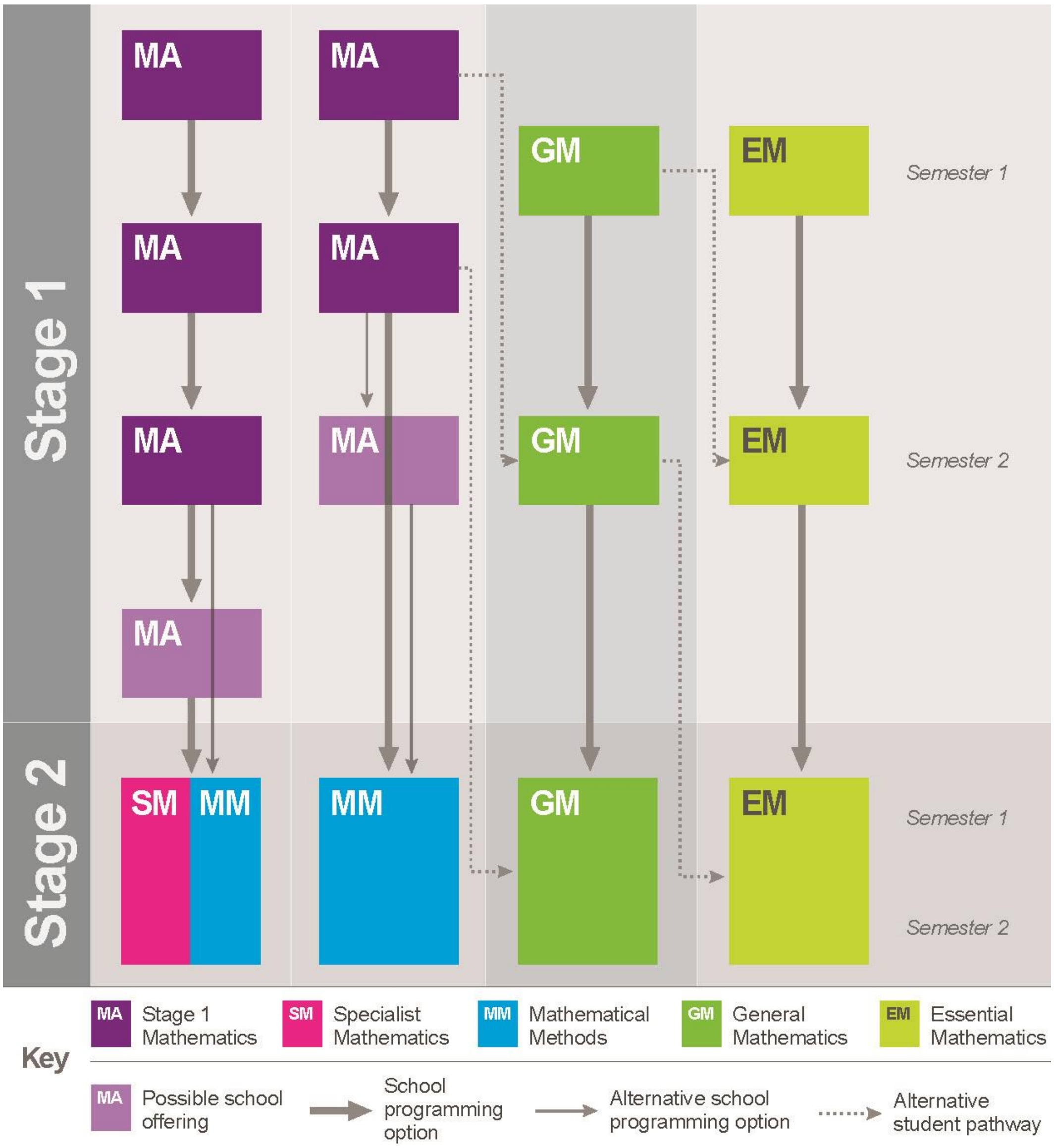
Specialist Mathematics draws on and deepens students' mathematical knowledge, skills, and understanding, and provides opportunities for students to develop their skills in using rigorous mathematical arguments and proofs, and using mathematical models. It includes the study of functions and calculus.

The subject leads to study in a range of tertiary courses such as mathematical sciences, engineering, computer science, and physical sciences. Students envisaging careers in related fields will benefit from studying this subject.

Specialist Mathematics is designed to be studied in conjunction with Mathematical Methods.

MATHEMATICAL OPTIONS

The diagram below represents the possible mathematical options that students might study at Stage 1 and Stage 2.



Notes:

Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum per se is not a prerequisite for the study of Specialist Mathematics and Mathematical Methods. The essential aspects of 10A are included in the relevant topics.

Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:

- literacy
- numeracy
- information and communication technology (ICT) capability
- critical and creative thinking
- personal and social capability
- ethical understanding
- intercultural understanding.

Literacy

In this subject students develop their literacy capability by, for example:

- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphical, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use mathematical skills, concepts, and technologies in a range of contexts that can be applied to:

- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- using spatial sense and geometric reasoning
- investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical and algebraic representations, and other mathematical models.

Information and communication technology (ICT) capability

In this subject students develop their information and communication technology capability by, for example:

- understanding the role of electronic technology in the study of mathematics
- making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant for study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice, and application; for example, to use data, address problems, and operate systems in particular situations.

Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- developing mathematical reasoning skills to think logically and make sense of the world
- understanding how to make and test projections from mathematical models
- interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

Ethical understanding

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- examining critically ways in which the media present particular perspectives
- sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols which have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of high-quality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 Mathematics with a C grade or better, or 20 credits of Stage 2 Specialist Mathematics with a C– grade or better, will meet the numeracy requirement of the SACE.

LEARNING SCOPE AND REQUIREMENTS

LEARNING REQUIREMENTS

The learning requirements summarise the key skills, knowledge and understanding that students are expected to develop and demonstrate through learning in Stage 2 Specialist Mathematics.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. think mathematically by posing questions, solving problems, applying models, and making, testing, and proving conjectures
4. interpret results, draw conclusions, and determine the reasonableness of solutions in context
5. make discerning use of electronic technology to solve problems and refine and extend mathematical knowledge
6. communicate mathematically and present mathematical information in a variety of ways.

CONTENT

Stage 2 Specialist Mathematics is a 20-credit subject.

The topics in Stage 2 extend students' mathematical experience and their mathematical flexibility and versatility, in particular, in the areas of complex numbers and vectors. The general theory of functions, differential equations, and dynamic systems provides opportunities to analyse the consequences of more complex laws of interaction.

Specialist Mathematics topics provide different scenarios for incorporating mathematical arguments, proofs, and problem-solving.

Stage 2 Specialist Mathematics consists of the following six topics:

- Topic 1: Mathematical induction
- Topic 2: Complex numbers
- Topic 3: Functions and sketching graphs
- Topic 4: Vectors in three dimensions
- Topic 5: Integration techniques and applications
- Topic 6: Rates of change and differential equations.

The suggested order of the topics is a guide only; however, students study all six topics.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns, as a series of key questions and key concepts, side by side with considerations for developing teaching and learning strategies.

The key questions and key concepts cover the prescribed content for teaching, learning, and assessment in this subject. The considerations for developing teaching and learning strategies are provided as a guide only.

A problem-based approach is integral to the development of the mathematical models and associated key concepts in each topic. Through key questions, teachers can develop the key concepts and processes that relate to the mathematical models required to address the problems posed.

The considerations for developing teaching and learning strategies present problems for consideration, and guidelines for sequencing the development of the concepts. They also give an indication of the depth of treatment and emphases required.

Although the material for the external examination will be based on the key questions and key concepts outlined in the six topics, the considerations for developing teaching and learning strategies may provide useful contexts for examination questions.

Students use electronic technology, where appropriate, to enable complex problems to be solved efficiently.

Topic 1: Mathematical induction

This topic builds on students' study of Subtopic 12.2: Introduction to mathematical induction in Stage 1 Mathematics. As students work through Stage 2 material, opportunities will arise to apply this method of proof in many contexts; for example, trigonometry, complex numbers, and matrices.

Subtopic 1.1: Proof by mathematical induction

Key questions and key concepts	Considerations for developing teaching and learning strategies
Proof by mathematical induction • Understand the nature of inductive proof, including the initial statement and inductive step	Formal proofs are expected, for example: Let there be associated with each positive integer n, a proposition $P(n)$. If $P(1)$ is true, and for all k, $P(k)$ is true implies $P(k+1)$ is true, then $P(n)$ is true for all positive integers n. • Prove divisibility results, such as $3^{2n+4}-2^{2n}$ is divisible by 5 for any positive integer n. • Prove results for sums of more complex series, such as $1.2+2.3+3.4+\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3}$ or $1.3+2.3^2+3.3^3+\cdots+n3^n=\frac{3}{4}\left((2n-1)3^n+1\right)$ • Prove other results, for example: $\begin{bmatrix}1&1\\0&1\end{bmatrix}^n=\begin{bmatrix}1&n\\0&1\end{bmatrix}$ for all positive integers n, or If $x\neq 1$, then $(1+x)(1+x^2)(1+x^4)\cdots\left(1+x^{2^{n-1}}\right)=\frac{1-x^{2^n}}{1-x}$ for all $n\in Z^+$, or $\cos x.\cos 2x.\cos 4x.\cdots\cos\left(2^{n-1}x\right)=\frac{\sin\left(2^n x\right)}{2^n\sin x}$ for all $n\in Z^+$ • Mathematical induction proofs that use inequalities and recursion formulae are not a requirement of this course

Topic 2: Complex numbers

The Cartesian form of complex numbers was introduced in Topic 12: Real and complex numbers in Stage 1 Mathematics, and the study of complex numbers is now extended to the polar form. The arithmetic of complex numbers is developed and their geometric interpretation as an expansion of the number line into a number plane is emphasised. Their fundamental feature — that every polynomial equation has a solution over the complex numbers — is reinforced and de Moivre's theorem is used to find n^{th} roots.

Subtopic 2.1: Cartesian and polar forms

Key questions and key concepts	Considerations for developing teaching and learning strategies
Understanding complex numbers involves: • real and imaginary parts, $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$, of a complex number • Cartesian form • arithmetic using Cartesian forms	
Is the Cartesian form always the most convenient representation for complex numbers? Is it the most convenient form for multiplying complex numbers?	Consider describing sets of points in the complex plane, such as circular regions or regions bounded by rays from the origin. For example: $\lvert z-\mathrm{i}\rvert \le 2,\ \arg(z)=\frac{\pi}{4}$
How to do arithmetic using polar form: • Conversion between Cartesian form and polar form	The conversions $x = r\cos\theta$, $y = r\sin\theta$, where $r=\sqrt{x^2+y^2}$ and $\tan\theta=\left(\frac{y}{x}\right)$, and $-\pi<\theta\le\pi$, along with $\cos\theta=\left(\frac{x}{\sqrt{x^2+y^2}}\right)$, and $\sin\theta=\left(\frac{y}{\sqrt{x^2+y^2}}\right)$, and their use in converting between Cartesian form and polar form. Calculators can be used, both to check calculations and enable students to consider examples that are not feasible by hand.
• The properties $\lvert z_1z_2\rvert=\lvert z_1\rvert\lvert z_2\rvert$ $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ $\mathrm{cis}\theta_1.\mathrm{cis}\theta_2=\mathrm{cis}(\theta_1+\theta_2)$ $\frac{\mathrm{cis}\theta_1}{\mathrm{cis}\theta_2}=\mathrm{cis}(\theta_1-\theta_2)$ They are the basis on which multiplication by $r\mathrm{cis}\theta$ is interpreted as dilation by *r* and rotation by θ	These properties make polar form the most powerful representation for dealing with multiplication. Students observe that the real and imaginary parts of the identity $\mathrm{cis}(\theta_1+\theta_2)=\mathrm{cis}\theta_1.\mathrm{cis}\theta_2$ are the addition of angles formulae for cosine and sine. Thus, complex multiplication encodes these trigonometric identities in a remarkable and simple way.

Key questions and key concepts	Considerations for developing teaching and learning strategies
• The utility of the polar form in calculating multiplication, division, and powers of complex numbers, and the geometrical interpretation of these	The properties $\lvert z_1 z_2 \cdots z_n \rvert = \lvert z_1 \rvert \lvert z_2 \rvert \cdots \lvert z_n \rvert$ and $\operatorname{cis}(\theta_1 + \theta_2 + \cdots \theta_n) = \operatorname{cis}\theta_1 . \operatorname{cis}\theta_2 \cdots \operatorname{cis}\theta_n$ could be argued by mathematical induction. The geometric significance of multiplication and division as dilation and rotation is emphasised, along with the geometric interpretation of modulus as distance from the origin. Topic 8: Geometry from Stage 1 Mathematics can be used; for example, using properties of a rhombus to determine the polar form of $z+1$ from that of z when z is on the unit circle.
• Prove and use de Moivre's theorem	Note also the special case of $\lvert z^n \rvert = \lvert z \rvert^n$, which can be verified by making all the zs equal in $\lvert z_1 z_2 \cdots z_n \rvert = \lvert z_1 \rvert \lvert z_2 \rvert \cdots \lvert z_n \rvert$. Also $\operatorname{cis}(n\theta) = (\operatorname{cis}\theta)^n$, which can be verified by making all the θs equal in $\operatorname{cis}(\theta_1 + \theta_2 + \cdots \theta_n) = \operatorname{cis}\theta_1 . \operatorname{cis}\theta_2 \cdots \operatorname{cis}\theta_n$. These two special cases combined form de Moivre's theorem for rational n. Prove this theorem by mathematical induction.
• Extension to negative integral powers and fractional powers	Extend to negative powers via $\operatorname{cis}(-n\theta) = \dfrac{1}{\operatorname{cis}(n\theta)} = \dfrac{1}{\operatorname{cis}(\theta)^n} = (\operatorname{cis}\theta)^{-n}$

Subtopic 2.2: The complex (Argand) plane

Key questions and key concepts	Considerations for developing teaching and learning strategies
What connections are there with previous study (e.g. Stage 1) on vectors and linear transformations in the plane?	
• Examine and use addition of complex numbers as vector addition in the complex plane	Correspondence between the complex number $a+b\mathrm{i}$, the coordinates $(a,\ b)$, and the vector $[a,\ b]$. Complex number addition corresponds to vector addition via the parallelogram rule. Complex numbers can be regarded as two-dimensional vectors with rules for multiplication and division, to supplement addition and subtraction.
• Examine and use multiplication as a linear transformation in the complex plane	Multiplying a complex number w by $z=r\operatorname{cis}\theta$ dilates w by a factor of r and rotates w through an angle of θ.
• Use multiplication by i as anticlockwise rotation through a right angle	Although this follows from the addition formula for $\operatorname{cis}\theta$ it can be demonstrated directly that: $\mathrm{i}(x+\mathrm{i}y)=-y+\mathrm{i}x$ shows that multiplication by i sends $(x,\ y)$ to $(-y,\ x)$, which can be shown to be the effect on the coordinates of rotating a point about the origin anticlockwise through a right angle. Another approach is to use polar form: $$\mathrm{i}\operatorname{cis}\theta=\operatorname{cis}\frac{\pi}{2}\operatorname{cis}\theta=\operatorname{cis}\left(\frac{\pi}{2}+\theta\right).$$
• Apply the geometric notion of $\lvert z-w\rvert$ as the distance between points in the plane representing them	
• Investigate the triangle inequality	The triangle inequality may be used when considering the distance between points in the complex plane. Extension to the sum of several complex numbers can be argued by mathematical induction.

Key questions and key concepts

- Apply geometrical interpretation and solution of equations describing circles, lines, rays, and inequalities describing associated regions; obtaining equivalent Cartesian equations and inequalities where appropriate

Considerations for developing teaching and learning strategies

Graphical solution of equations and inequalities such as

$|z| < 2,\ |z-3\mathrm{i}| = 4,\ |z+\mathrm{i}| = |z-1|,\ |z-1| = 2|z-\mathrm{i}|$,

$\theta = \frac{\pi}{4},\ \frac{\pi}{4} \le \arg z \le \frac{3\pi}{4},$

$\mathrm{Re}(z) > \mathrm{Im}(z),\ 0 < \mathrm{Im}(z) < 1$

strengthen the geometric interpretation.

An important aim is the conversion of such equations and inequalities into Cartesian form in the case of circles and lines, through geometric understanding of the descriptions used above directly in terms of modulus and argument. Dynamic geometry software can be used as an aid. Students can investigate various polar graphs, using graphing technology.

Subtopic 2.3: Roots of complex numbers

Key questions and key concepts	Considerations for developing teaching and learning strategies
Can polar form be used to efficiently find all solutions of the simplest n^{th} degree polynomial? • Solution of $z^n = c$ for complex c but in particular the case $c = 1$ • Finding solution of n^{th} roots of complex numbers and their location in the complex plane	As a particular example of the use of de Moivre's theorem, recognise the symmetric disposition of the n^{th} roots of unity in the complex plane. The fact that their sum is zero can be linked in the vector section of the subject outline to the construction of a regular n-gon.

Subtopic 2.4: Factorisation of polynomials

Key questions and key concepts	Considerations for developing teaching and learning strategies
Operations on real polynomials: polynomials can be added and multiplied	Briefly consider the multiplication process: the division algorithm using long division or synthetic division, or the multiplication process with inspection.
What can be said about division of polynomials?	Discuss the use of undetermined coefficients and equating coefficients in factoring when one factor is given.
• Consider roots, zeros, and factors • Prove and apply the factor and remainder theorem; its use in verifying zeros	Explore and understand the correspondence between the roots of a polynomial equation, the zero of a polynomial, and the linear factor of a polynomial.
• Consider conjugate roots in factorisation of cubics and quartics with real coefficients (given a zero)	Use conjugate pairs to create a real quadratic factor of a given real polynomial.
• Solve simple real polynomial equations	Real polynomials can be factored into real linear and quadratic factors, and into linear factors with the use of complex numbers. Explore the connection between the zeros and the shape and position of the graph of a polynomial. There are many opportunities to make use of graphing technology. Use special examples: $x^n = 1$ or -1 as solved above by de Moivre's theorem; the factorisation of $x^3 \pm a^3$, $x^4 \pm a^4$ and so on, as an illustration of the use of the remainder theorem. The statement of the fundamental theorem can be considered to answer the question: Why were complex numbers 'invented'? Though not every real polynomial of degree *n* has *n* real zeros, in the field of complex numbers every real or complex polynomial of degree *n* has exactly *n* zeros (counting multiplicity).

Topic 3: Functions and sketching graphs

The study of functions and techniques of graph sketching, introduced in Stage 1 Mathematics, is extended and applied in the exploration of inverse functions and the sketching of graphs of composite functions involving absolute value, reciprocal, and rational functions.

Subtopic 3.1: Composition of functions

Key questions and key concepts	Considerations for developing teaching and learning strategies
How and when can new functions be built from other functions?	
• If *f* and *g* are two functions, then the composition function $(f \circ g)(x)$ is defined by $f(g(x))$ if this exists	
• Determine when the composition $(f \circ g)(x) = f(g(x))$ of two functions is defined	The composition is defined for those values of *x* for which $g(x)$ is in the domain of *f*. For example: given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3x$, $f(g(x)) = \sqrt{x^2 - 3x}$. Since $x \geq 0$ is the domain of $f(x)$, $g(x) = x^2 - 3x \geq 0$ for $f(g(x))$ to exist. Hence $x \leq 0$ or $x \geq 3$. Equivalently, $f \circ g$ is defined when the range of *g* is contained in the domain of *f*.
• Find compositions	Using examples of known functions, obtain such compositions as $\sqrt{(\sin x)}$ and $\frac{1}{x-3}$ with appropriate domains.

Subtopic 3.2: One-to-one functions

Key questions and key concepts	Considerations for developing teaching and learning strategies
When is it possible for one function to 'undo' another function? Consider functions that are one-to-one:	
• Determine if a function is one-to-one	Function f is one-to-one if $f(a)=f(b)$ only when $a=b$. The vertical line test (used to determine whether a relation is a function) is mentioned in Subtopic 1.4: Functions, Stage 1 Mathematics. A further property of functions that can be applied is that a one-to-one function has an inverse. The horizontal line test can be used to provide some evidence that a function is a one-to-one function. If the graph of a function fails the horizontal line test, then the function is not one-to-one and will not have an inverse.
The inverse f^{-1} of a one-to-one function • Determine the inverse of a one-to-one function	Appreciate that an inverse function for f can only be defined when each element of the range corresponds to a unique value in the domain. For example, $y=f(x)=2x+3$, then $x=\frac{(y-3)}{2}$, so $f^{-1}(x)=\frac{(x-3)}{2}$.
Relationship between the graph of a function and the graph of its inverse • Investigate symmetry about $y=x$	Examples include the pair $2x+3$, $\frac{x-3}{2}$, and (for non-negative x) the pair $\sqrt{x}$, x^2. Also $\sin x$, for $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ and its inverse $\arcsin x$ for $-1\le x\le 1$. Dynamic geometry software and graphics calculators can be used to investigate these relationships. (Note that the exponential and logarithmic functions and their inverse relationship are studied in Stage 2 Mathematical Methods.)

Subtopic 3.3: Sketching graphs

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can available information be put together to deduce the behaviour of various functions that are composite functions?	
• Absolute value function and its properties	Use and apply the notation $\|x\|$; the graph of $y=\|x\|$.
• Compositions involving absolute values and reciprocals	If $f(x)$ is some given function, investigate the relationship between the graphs of $y=f(x)$ and the graphs of these compositions: $y=\frac{1}{f(x)}$, $y=\|f(x)\|$, $y=f(\|x\|)$. Note: when referring to $\frac{1}{f(x)}$, $f(x)$ is linear, quadratic, or trigonometric.
• Graphs of rational functions	Sketch the graphs of rational functions where the numerator and denominator are polynomials of up to degree 2 with real zeros. Students may use technology to determine the behaviour of the function near the asymptotes.

Topic 4: Vectors in three dimensions

The study of vectors was introduced in Stage 1 Mathematics with a focus on vectors in two-dimensional space. Three-dimensional vectors are now introduced, enabling the study of lines and planes in three dimensions, their intersections, and the angles they form. Further development of vector methods of proof enables students to solve geometric problems in three dimensions.

Students gain an understanding of the interrelationships of Euclidean, vector, and coordinate geometry, and appreciate that the proof of a geometric result can be approached in different ways.

Subtopic 4.1: The algebra of vectors in three dimensions

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is a vector?	Briefly consider vectors as directed line segments (with arrows) in space, generalising from the two-dimensional treatment in Topic 9: Vectors in the plane, Stage 1 Mathematics, including unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$.

Subtopic 4.2: Vector and Cartesian equations

Key questions and key concepts	Considerations for developing teaching and learning strategies
How are points represented in three-dimensional space?	
• Introduce Cartesian coordinates by plotting points and considering relationships between them	Investigate examples such as vertical and horizontal planes, planes as perpendicular bisectors and the equations of spheres. Teaching and learning in this topic can be enhanced with the use of current, free, three-dimensional dynamic geometry software such as Winplot, Geogebra.
How is the equation of a line in two and three dimensions written? • Consider vector, parametric, and Cartesian forms	Geometric considerations lead to the vector equation of a line. From this it is possible to derive the parametric form and the Cartesian form. Exercises highlight the construction of parallel lines, perpendicular lines, and the phenomenon of skewness. Compute the point of a given line that is closest to a given point; distance between skew lines; and angle between lines.
Can it be determined whether or not the paths of two particles cross?	Examine the position of two particles each described as a vector function of time, and determine whether their paths cross or the particles meet.
Can vectors be multiplied together? What is the meaning of the product? • Scalar (dot) product and vector (cross) product. What are their properties? Interpret them in context	These two operations on pairs of vectors provide important geometric information. The scalar product (extended into three dimensions) and the vector product are treated in terms of coordinates, and length and angle. There are conditions for perpendicularity and parallelism, and construction of perpendiculars.
• Perform cross-product calculation using the determinant to determine a vector normal to a given plane	2×2 determinants are treated in Topic 11: Matrices, Stage 1 Mathematics.
• $\lvert \boldsymbol{a}\times\boldsymbol{b} \rvert$ is the area of the parallelogram with sides $\boldsymbol{a}$ and $\boldsymbol{b}$	The cross-product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in three dimensions is a vector, mutually perpendicular to $\boldsymbol{a}$ and to $\boldsymbol{b}$, whose length is the area of the parallelogram determined by $\boldsymbol{a}$ and $\boldsymbol{b}$. The right-hand rule determines its sense. The components and the vector itself may be expressed using determinants.
How can a plane be described by an equation in both vector and Cartesian form?	Derive the equation of a plane in Cartesian form, $Ax+By+Cz+D=0$. This form is developed using the dot product of a vector in the plane and a normal to the plane.

Key questions and key concepts	Considerations for developing teaching and learning strategies
What relationships between lines and planes can be described?	Students explore the following relationships: intersection of a line and a plane, angle between a line and a plane, and lines parallel to or coincident with planes. Students find the coordinates of a point on a given plane $Ax+By+Cz+D=0$ that is closest to a given point (x_1, y_1, z_1) in space. The distance between these two points is given by the formula: $\frac{\lvert Ax_1+By_1+Cz_1+D \rvert}{\sqrt{A^2+B^2+C^2}}$.
Prove geometric results in the plane and construct simple proofs in three dimensions: • Equality of vectors • Coordinate systems and position vectors; components • The triangle inequality	Vectors are equal if they form opposite sides of a parallelogram. Applications (e.g. navigation and force) as encountered in Topic 9: Vectors in the plane, Stage 1 Mathematics can be extended readily to three dimensions. Students note the previous application of the triangle inequality in Subtopic 2.2.
• The use of vector methods of proof, particularly in establishing parallelism, perpendicularity, and properties of intersections	Suitable examples include: • the angle in a semicircle is a right angle • medians of a triangle intersect at the centroid. The result: If $k_1\boldsymbol{a}+k_2\boldsymbol{b}=l_1\boldsymbol{a}+l_2\boldsymbol{b}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are not parallel, then $k_1=l_1$ and $k_2=l_2$. This is a result which can be proved by contradiction. Students consider some examples of proof by vector methods, and appreciate their power.

Subtopic 4.3: Systems of linear equations

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can a system of linear equations be solved?	Recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination (row operations) on augmented matrix form to solve a system of up to 3×3 linear equations.
For a system of equations with three variables, what solutions are possible and what is their geometric interpretation?	Discuss intersections of planes: algebraic and geometric descriptions of unique solution, no solution, and infinitely many solutions. Finding the intersection of a set of two or more planes amounts to solving a system of linear equations in three unknowns.

Topic 5: Integration techniques and applications

Integration techniques developed in Topic 3 of Stage 2 Mathematical Methods are extended to a greater range of trigonometric functions and composite functions, using inverse trigonometric functions and integration by parts. These techniques are applied to finding the areas between curves and the volumes of solids of revolution.

Subtopic 5.1: Integration techniques

Key questions and key concepts	Considerations for developing teaching and learning strategies
What techniques can be used to enable integration of a wider class of functions?	
• Use identities to simplify integrals of squared trigonometric functions	$\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$, $1+\tan^2 x = \sec^2 x$
• Use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))g'(x)$	For example, $\int \sin^2 x \cos x \mathrm{d}x$
• Use the formula $\int \frac{1}{x}\mathrm{d}x = \ln\lvert x\rvert + c$ for $x \neq 0$	Compare the case $x > 0$
How can use of the inverse trigonometric functions enable integration of certain functions?	This will involve discussion of restriction of domain in order to obtain a one-to-one function.
• Find and use the inverse trigonometric functions: arcsine, arccosine, arctangent	For example, the principal domain of $\sin x$ is the closed interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. (See also Subtopic 3.2: One-to-one functions.) For example, if $y = \arcsin(x)$, then $x = \sin y$.
• Find and use the derivatives of these functions • Hence integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$, $\frac{a}{a^2 + x^2}$	Use implicit differentiation and trigonometric identities to obtain the results.
• Use partial fractions for integrating rational functions in simple cases	For example: Verify that $\frac{1}{x+2} - \frac{1}{x-1} = \frac{-3}{(x+2)(x-1)}$ and hence find $\int \frac{1}{x^2+x-2}\mathrm{d}x$
• Use integration by parts	For integrals that can be expressed in the form $\int f'(x)g(x)\mathrm{d}x$, making use of the formula for differentiating a product: $\int f'(x)g(x)\mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\mathrm{d}x$ For example, $\int xe^x \mathrm{d}x$.

Subtopic 5.2: Applications of integral calculus

Key questions and key concepts	Considerations for developing teaching and learning strategies
What are some applications of the integration techniques?	
• Areas between curves determined by functions	If $f(x) \geq g(x)$ for $a \leq x \leq b$, then the area of the region bounded by the graphs of the functions between $x = a$ and $x = b$ is $\int_a^b (f(x) - g(x))\mathrm{d}x.$
• Volumes of solids of revolution about either axis	Let $y = f(x)$ be non-negative on the interval $[a, b]$. The volume of the solid that is obtained when the region of the graph of the function that is bounded by the lines $x = a$ and $x = b$ is rotated about the *x*-axis is given by $V = \int_a^b \pi y^2 \mathrm{d}x.$ If $y = f(x)$ is one-to-one, and is positive on the interval $[a, b]$, then the volume of the solid that is obtained when the region of the graph of $x = f^{-1}(y)$ that is bounded by the lines $y = f(a) = c$ and $y = f(b) = d$ is rotated about the *y*-axis is $V = \int_c^d \pi x^2 \mathrm{d}y$. Derivation of these formulae can be investigated through graphical approaches.

Topic 6: Rates of change and differential equations

A mathematical equation that relates a function to its derivatives is known as a differential equation. In applications, functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two.

This topic continues the study of differentiation and integration of functions. Calculus techniques are applied to vectors and simple differential equations. The study of rates of change and differential equations demonstrates applications of learning throughout this subject, in a range of contexts.

This topic also highlights the fundamental theorem of calculus from the point of view of differential equations, intended to deepen students' perspective on indefinite integrals.

Subtopic 6.1: Implicit differentiation

Key questions and key concepts	Considerations for developing teaching and learning strategies
Can information be found about the derivative of a function even when there is no explicit formula for it in the form $y = f(x)$?	Following on from the rules of differentiation studied in Stage 2 Mathematical Methods, implicit differentiation is used to find the gradient of curves in implicit form. For example: Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $x^3 + xy = 5e^y$.
• Implicit differentiation	Knowledge of implicit differentiation is required to justify the derivative of the logarithm function. That is: If $y = \ln x$ then $e^y = x$. Consider $e^y = x$ and differentiate both sides with respect to x to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$.

Subtopic 6.2: Differential equations

<table>
<tr><th>Key questions and key concepts</th><th>Considerations for developing teaching and learning strategies</th></tr>
<tr><td>What is the relationship between the rates of change of two related functions of time?
• Related rates</td><td>Where two functions of time $x(t)$ and $y(t)$ are related by $y(t)=f(x(t))$, their rates of change are related by the chain rule
$y'(t)=f'(x(t))\times x'(t)$.
An example of calculating $y'(t)$ from $x'(t)$ and the derivative of $f(x(t))$
• $x(t)$ might be the length of the side of a square that is changing over time, and $y(t)$ its area
• The function $f(x(t))$ relates the length of the side of a square to its area.</td></tr>
<tr><td>What is a differential equation?</td><td>A differential equation is an equation that expresses a relationship between a function and its rates of change.</td></tr>
<tr><td>• Solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=f(x)$
• Solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=f(x)g(y)$</td><td>The simplest differential equation is of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=f(x)$, which may be solved by integration techniques. Differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=g(y)$ and in general form $\frac{\mathrm{d}y}{\mathrm{d}x}=f(x)g(y)$ may be solved using separation of variables.</td></tr>
<tr><td>How can the information about the derivative of a function be described?</td><td>An equation $y'(x)=f(x)$ indicates the slope of the graph at each point x but not the value of y. A line of gradient $f(x_0)$ can be drawn at each point on each vertical line $x=x_0$ and one of these is the tangent line to the graph. This family of lines, one through each point in the plane, is called the 'slope field'.</td></tr>
<tr><td>• Examine slope fields of first-order differential equations
• Reconstruct a graph from a slope field both manually and using graphics software</td><td>Given an initial value, say, y_0 for y at $x=x_0$ the slope field indicates the direction in which to draw the graph. These concepts should be displayed using graphics calculators or software. Students consider how the computer or graphics calculator might have traced the curves — for example, by following each slope line for a small distance, then following the slope line at the new point.</td></tr>
</table>

Key questions and key concepts	Considerations for developing teaching and learning strategies
	Compare these graphical results with known solutions from integration, such as the solutions $y = x^2 + c$ for the equation $y'(x) = 2x$. What role does c play in the geometric picture? What is the key feature of a family of curves of the form $F(x) + c$?
How can differential equations be used in modelling? • Formulate differential equations in contexts where rates are involved • Use separable differential equations	The exponential equation $y'(x) = ky(x)$ is the simplest example of a separable differential equation. Other examples, such as $y' = k(A - y)$, arise from Newton's law of cooling or as models of the spread of rumours. They are solved in terms of exponential functions. Emphasise the family of solutions and the use of initial conditions to determine which one describes a problem.
• Use the logistic differential equation	This logistic differential equation $y' = k(P - y)y$ is interpreted in terms of a population P (of molecules, of organisms, etc.), of which y are active or infected and $P - y$ are not. To carry out the method for solving the logistic differential equation, students check the identity $\frac{1}{y} + \frac{1}{P - y} = \frac{P}{y(P - y)}$.

Subtopic 6.3: Pairs of varying quantities — polynomials of degree 1 to 3

Key questions and key concepts	Considerations for developing teaching and learning strategies
What curves are traced out by a moving point $(x(t),\ y(t))$ in which the functions $x(t)$ and $y(t)$ are polynomials of degree 1 to 3?	
Consider examples of applications to: • uniform motion • vector interpretation	Being uniform, the quantities are functions of the form $x(t)=x_0+at,\ y(t)=y_0+bt$. These describe the parametric equation of a line in two dimensions. This is understood with reference to the vector and parametric equations of lines in three dimensions. The vector form of the two-dimensional equation is $[x_0+ta,\ y_0+tb]=[x_0,\ y_0]+t[a,\ b]$.
• objects in free flight	The position of an object in free flight is given by the equations $$x(t)=x_0+at,\ y(t)=y_0+bt-\frac{1}{2}gt^2,$$ where $(x_0,\ y_0)$ is the initial position, $[a,\ b]$ is the initial velocity, and g is the acceleration due to gravity. Students observe the parabolic shape of the curve. This equation can be used to answer questions such as: At what angle should a netball be thrown to score a goal? At what angle should a cricket ball be struck to clear the fence? A further example of an application: • Degree 3 polynomials where $0\le t\le 1$ feature in computer-aided design. The use of Bézier curves attempts to mimic freehand drawing using cubic polynomials, and motion along Bézier curves is used to simulate motion in computer animations. They are constructed using four control points, two marking the beginning and end of the curve and two others controlling the shape. They can be constructed with interactive geometry software such as that available from www.moshplant.com/direct-or/bezier/.

Subtopic 6.4: Related rates, velocity, and tangents

Key questions and key concepts	Considerations for developing teaching and learning strategies
How is the motion of a moving point described? • For a moving point $(x(t),\ y(t))$, the vector of derivatives $\boldsymbol{v}=[x'(t),\ y'(t)]$ is naturally interpreted as its instantaneous velocity	For uniform motion, the velocity $[a,\ b]$ has for its components the rates of change of the components of the position vector. A logical extension of this concept, in which average rates of change are replaced by instantaneous rates of change, or derivatives, is that $\boldsymbol{v}=[x'(t),\ y'(t)]$ is interpreted as the instantaneous velocity vector of the moving point $(x(t),\ y(t))$.
The Cartesian equation of the path of the moving point can be found by eliminating t and establishing the relationship between y and x.	If, given a function $f(x)$ and a function of time $x(t)$, you set $y(t)=f(x(t))$, then the moving point $(x(t),\ y(t))$ travels along the graph of $f(x)$.
• The velocity vector is always tangent to the curve traced out by a moving point	The chain rule $y'(t)=f'(x(t))\times x'(t)$ shows that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}=f'(x(t))$ and hence that the velocity vector is a tangent to the graph. Since effectively every curve is the graph of a function, the velocity vector is always tangent to the curve traced out by a moving point.
• The speed of the moving point is the magnitude of the velocity vector, that is, $\sqrt{x'^2(t)+y'^2(t)}=\sqrt{\boldsymbol{v}\bullet\boldsymbol{v}}$	Calculate the speed of projectiles or of points moving along parameterised curves.
• Find the arc length along parametric curves	$\boldsymbol{v}=\frac{\mathrm{d}}{\mathrm{d}t}[x(t),\ y(t)]$ is the velocity vector of the moving point as it traces out the curve. If the point moves during the time from $t=a$ to $t=b$, the length of the path traced out is calculated using the integral $l=\int_a^b\sqrt{\boldsymbol{v}\bullet\boldsymbol{v}}\,\mathrm{d}t$.

Subtopic 6.5: Trigonometric parameterisations

Key questions and key concepts	Considerations for developing teaching and learning strategies
• A point moving with unit speed around the unit circle can be described using the moving position vector $\boldsymbol{P}(t)=[\cos t,\ \sin t]$	Students can find the first and second derivatives of $\boldsymbol{P}(t)$ and establish the constant speed and centripetal acceleration of circular motion.
• Consider the speed of moving around other circles with other speeds	The more general position vector $\boldsymbol{P}(t)=[R\cos\omega t,\ R\sin\omega t]$ allows for faster or slower motion around smaller or larger circles. Students can use the arc length formula to establish the well-known formula for the circumference of a circle.
• Other forms of parametric equations $\boldsymbol{P}(t)=[x(t), y(t)]$ where $x(t)$ and $y(t)$ are trigonometric functions that may not result in circular motion	For example, a moving particle's position may be given by $x(t)=3\cos t$ $y(t)=2\sin 2t$ which results in a non-circular path.

ASSESSMENT SCOPE AND REQUIREMENTS

All Stage 2 subjects have a school assessment component and an external assessment component.

EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 2 Specialist Mathematics.

School assessment (70%)

- Assessment Type 1: Skills and Applications Tasks (50%)
- Assessment Type 2: Mathematical Investigation (20%)

External assessment (30%)

- Assessment Type 3: Examination (30%)

Students provide evidence of their learning through eight assessments, including the external assessment component. Students complete:

- six skills and applications tasks
- one mathematical investigation
- one examination.

ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by:

- teachers to clarify for students what they need to learn
- teachers and assessors to design opportunities for students to provide evidence of their learning at the highest possible level of achievement.

The assessment design criteria consist of specific features that:

- students should demonstrate in their learning
- teachers and assessors look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole, must give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships.

CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.

CT3 Application of mathematical models.

CT4 Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

The specific features are as follows:

RC1 Interpretation of mathematical results.

RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.

RC3 Use of appropriate mathematical notation, representations, and terminology.

RC4 Communication of mathematical ideas and reasoning to develop logical arguments.

RC5 Development, testing, and proof of valid conjectures.*

* In this subject students must be given the opportunity to develop, test, and prove conjectures in at least one assessment task in the school assessment component.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete six skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

In the remaining skills and applications tasks, electronic technology and up to one A4 sheet of handwritten notes (on one side only) may be used at the discretion of the teacher.

Students find solutions to mathematical problems that may:

- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine, analytical, and/or interpretative problems. Some of these problems should be set in context; for example, social, scientific, economic, or historical.

Students provide explanations and arguments, and use correct mathematical notation, terminology, and representations throughout the task.

Skills and applications tasks may provide opportunities to develop, test, and prove conjectures.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

Assessment Type 2: Mathematical Investigation (20%)

Students complete one mathematical investigation.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers should give some direction about the appropriateness of each student's choice and guide and support students' progress in an investigation. For this investigation there must be minimal teacher direction and teachers must allow the opportunity for students to extend the investigation in an open-ended context.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. computer algebra systems, spreadsheets, statistical packages) to enhance their investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, evidence of technological skills, and results are important considerations.

Students complete a report for the mathematical investigation.

In the report, students interpret and justify results, draw conclusions, and give appropriate explanations and arguments. The mathematical investigation may provide an opportunity to develop, test, and prove conjectures.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of the investigation report may be written or multimodal.

The investigation report, excluding bibliography and appendices if used, must be a maximum of 15 A4 pages if written, or the equivalent in multimodal form. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as two A4 pages reduced to fit on one A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination (30%)

Students undertake a 3-hour external examination.

The examination is based on the key questions and key concepts in the six topics. The considerations for developing teaching and learning strategies are provided as a guide only, although applications described under this heading may provide contexts for examination questions.

The examination consists of a range of problems, some focusing on knowledge and routine skills and applications, and others focusing on analysis and interpretation. Some problems may require students to interrelate their knowledge, skills, and understanding from more than one topic. Students provide explanations and arguments, and use correct mathematical notation, terminology, and representations throughout the examination.

A formula sheet is included in the examination booklet.

Students may take two unfolded A4 sheets (four sides) of handwritten notes into the examination room.

Students may use approved electronic technology during the external examination. However, students need to be discerning in their use of electronic technology to find solutions to questions/problems in examinations.

All specific features of the assessment design criteria for this subject may be assessed in the external examination.

PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers and assessors refer to in deciding how well students have demonstrated their learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of each school assessment type, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- assigning a grade between A+ and E– for the assessment type.

The student's school assessment and external assessment are combined for a final result, which is reported as a grade between A+ and E–.

Performance Standards for Stage 2 Specialist Mathematics

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures, with proof.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Some development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly effective development and testing of valid conjectures, with substantial attempt at proof.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures, with some attempt at proof.

	Concepts and Techniques	Reasoning and Communication
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness, or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.

ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement, in both the school assessment and the external assessment, are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 2 are available on the SACE website (www.sace.sa.edu.au)

SUPPORT MATERIALS

SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).