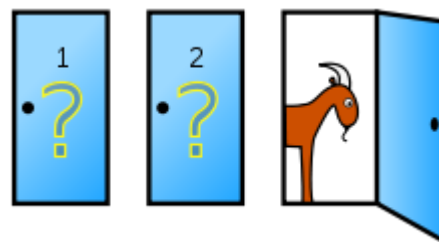


Monte Hall Problem

The Monte Hall problem is a classic example of how counter-intuitive probability can be. It is loosely based on the 1970's TV show "Let's make a deal", and is named after that show's original host: Monte Hall. The problem is traditionally represented as follows:

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?



Playing the Game:

You may have an opinion on the Monte Hall problem, but regardless the answer can be demonstrated by playing the game. We will play the Monte Hall problem, but we will use substitutes for the doors, the goats, and the car.

- Organise yourselves into pairs, and assign roles: one host, one player.
- The host puts three tokens (two "goats" and one "car") under three identical cups (the "doors", noting which "door" has the "car" behind it.
- The host then mixes the cups around, taking care to remember where the "car" is, while the player looks away.
- The player chooses a cup.
- The host reveals one of the cups the player did not choose with a "goat" under it.
- The player either chooses to keep their choice, or switch to the other cup.
- The host reveals the outcome: if the player chose the cup with the "car" under it, they win, if they chose a "goat", they lose.
- Play the game several times, and record how many times the player wins and loses using each method (keeping the original, or switching to the other cup).
- Switch roles and repeat. If you like, one of you can always keep the original choice and the other can always switch, or do a combination.

Solution:

It is better to switch cups. It might seem, intuitively, that it shouldn't, but the math checks out. The argument is as follows: your original choice has a $\frac{1}{3}$ probability of being the car. Hence, the other cup must make up the remaining probability: $\frac{2}{3}$. One of the subtle keys here is that the host knows which cup has the car in it, so if you did not choose the car originally, they won't accidentally reveal the car. To further convince yourself or demonstrate the principle, try playing the game with some additional "goats". If you had 10 doors with 9 goats and 1 car behind them, and once you picked one the host revealed 8 doors with goats behind them, should you stick to your original choice, or switch? If you test this version with the cups you will find it dramatically demonstrates the principle.