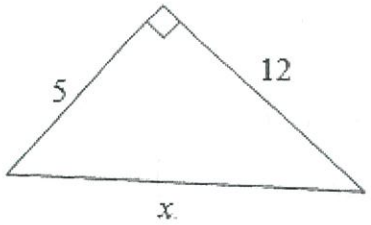
Stage 1 - General Mathematics Trigonometry Test

- Unless otherwise stated, give all answers to 1 decimal place

1 Find the value of x for each of the triangles below:

a

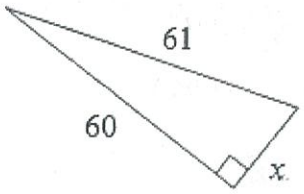


a) $x = \sqrt{5^2 + 12^2}$

$= \sqrt{169}$

$= 13.0$

b



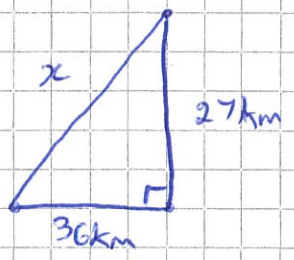
b) $x = \sqrt{61^2 - 60^2}$

$= \sqrt{121}$

$= 11.0$

Notice x is not the hypotenuse, which means you need to use the subtraction version of pythag

2 A helicopter flies due south for 27 km, then turns and heads due west for 36 km. How far is the helicopter from its starting point?



$x = \sqrt{27^2 + 36^2}$

$= \sqrt{2025}$

$= 45\text{ km}.$

3 A triangle has sides of 15 cm, 25 cm and 30 cm.

a) Show that this triangle is not right angled

$15^2 + 25^2 = 850$

$30^2 = 900.$

So $15^2 + 25^2 \neq 30^2$

so the triangle is not right-angled.

b) Find the area of this triangle

semi perimeter

$s = \frac{15 + 25 + 30}{2} = 35.$

Area $= \sqrt{35(35-15)(35-25)(35-30)}$

$= \sqrt{35 \times 20 \times 10 \times 5}$

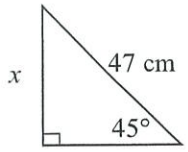
$= \sqrt{35000}$

$\approx 187.1\text{ cm}^2.$

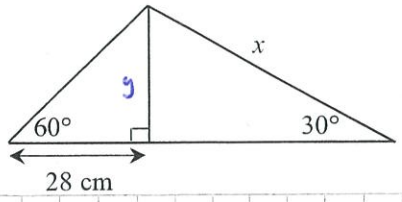
Using pythag to check this is just one way to answer this. There are other ways too (using the cosine rule for example)

4 Find the value of x in each of the following.

a



b



edit CAH TOA.

a) $\sin(45°) = \frac{x}{47}$

$x = 47 \times \sin(45)$

$= 47/\sqrt{2}$

$\approx 33.2\text{ cm}$

b) $\tan(60°) = \frac{y}{28}$

$y = 28 \times \tan(60)$

$\approx 48.5\text{cm}$

$\sin(30°) = \frac{y}{x}$

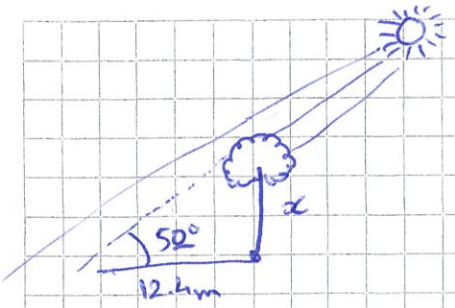
$x = \frac{y}{\sin(30°)}$

$= \frac{48.5}{\sin(30°)}$

$\approx 97.0\text{ cm}$

two-step process.

5 From the horizon, the sun is at an angle of elevation 52°. Find the height of a tree which casts a shadow of 12.4m.



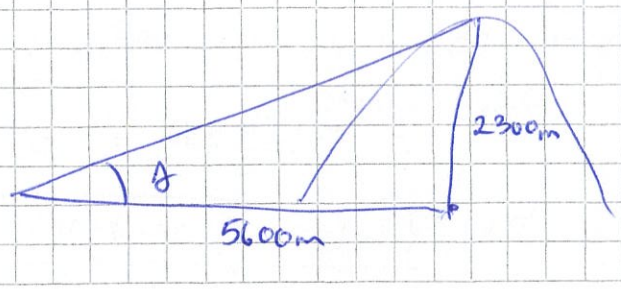
$\tan(52°) = \frac{x}{12.4}$

$x = 12.4 \times \tan(52°)$.

$\approx 15.9\text{m}$.

diagram of then SOH CAH TOA

6 Find the angle of elevation to the top of a mountain 2300m high from a point 5.6km from its base.



$\tan(\theta) = \frac{2300}{5600}$

$\theta = \tan^{-1}\left(\frac{2300}{5600}\right)$

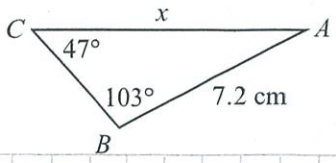
$\approx 22.3°$

Same here, diagram then SOH CAH TOA.

trick is to make sure you convert the lengths to be in the same units, doesn't matter if its m or km, works either way,

7 Use the sine rule to find the value of x. Give your answers correct to two decimal places.

a

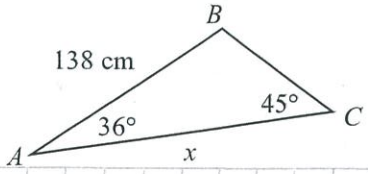


$$\frac{\sin(103°)}{x} = \frac{\sin(47°)}{7.2}$$

$$x = \frac{7.2 \times \sin(103)}{\sin(47)}$$

$$\approx 9.59 \text{ cm}.$$

b



$$\angle ABC = 180 - 36 - 45$$
$$= 99°$$

$$\frac{\sin(99)}{x} = \frac{\sin(45)}{138}$$

$$x = \frac{138 \times \sin(99)}{\sin(45)}$$

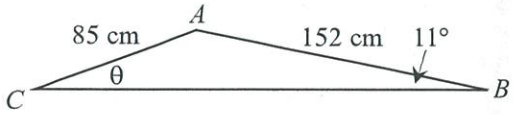
$$\approx 192.76 \text{ cm}$$

Sine rule (for a side length)

Trick is you need to realise you're missing angle.

8 Use the sine rule to find the value of θ, to the nearest degree.

a

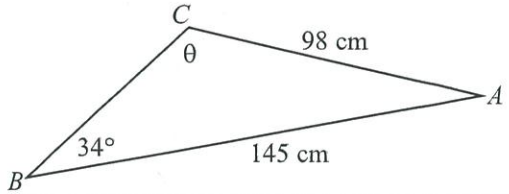


$$\frac{\sin(11)}{85\text{cm}} = \frac{\sin(\theta)}{152\text{cm}}$$

$$\theta = \sin^{-1}\left(\frac{152 \times \sin(11)}{85}\right)$$

$$\approx 20°$$

b



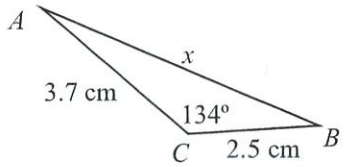
$$\frac{\sin\theta}{145} = \frac{\sin(34)}{98}$$

$$\theta = \sin^{-1}\left(\frac{145 \times \sin(34)}{98}\right)$$

$$\approx 156°$$

Sine rule (for an angle)

9 Use the cosine rule to find the value of x. Give your answer correct to one decimal place.



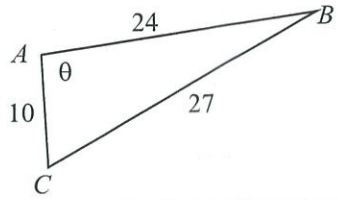
Cosine rule (for a side length)

$x = \sqrt{3.7^2 + 2.5^2 - 2 \times 3.7 \times 2.5 \cos(134)}$

$= \sqrt{32.791}$

$\approx 5.7\text{cm}$

10 Find the value of θ to the nearest degree.



Cosine rule (for an angle)

$\theta = \cos^{-1}\left(\frac{10^2 + 24^2 - 27^2}{2 \times 10 \times 24}\right)$

$\approx 96°$

11 If a triangle has side lengths 23 cm, 28 cm and 32 cm, find the size of the smallest angle, correct to one decimal place.

$a = 23$cm $\quad b = 28$cm $\quad c = 32$cm

$A = \cos^{-1}\left(\frac{28^2 + 32^2 - 23^2}{2 \times 28 \times 32}\right) \approx 44.5°$

$B = \cos^{-1}\left(\frac{23^2 + 32^2 - 28^2}{2 \times 23 \times 32}\right) \approx 58.5°$

$C = \cos^{-1}\left(\frac{23^2 + 28^2 - 32^2}{2 \times 23 \times 28}\right) \approx 77.0°$.

you need cosine rule for calculating the angles here, and you can either calc all three angles, OR you can use this trick to realise the smallest angle will be opposite the smallest side length.

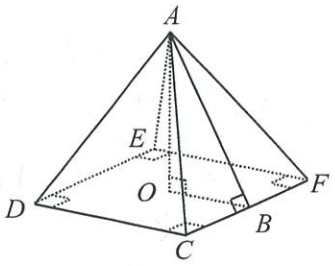
12 In the square-based pyramid shown, $AO = 12$ cm, $DC = 10$ cm and B is the midpoint of CF. Find:

a the exact value of the length AB

b the angle $\angle OBA$ (correct to one decimal place)

c the length AF (correct to one decimal place)

d the angle $\angle AFB$ (correct to one decimal place)



this involves some 3D thinking. drawing diagrams for each of the triangles involved could be helpful.

$OB = \frac{1}{2} DC = \frac{1}{2}(10) = 5$cm $\quad AO = 12$cm.

so $AB = \sqrt{5^2 + 12^2}$

$= \sqrt{169}$

$= 13$cm

$\angle OBA = \cos^{-1}\left(\frac{5^2 + 13^2 - 12^2}{2 \times 5 \times 13}\right)$ $\quad \angle OBA = \tan^{-1}\left(\frac{12}{5}\right)$

$\approx 67.4°$ $\quad \approx 67.4°$

$CF = DC = 10$cm, so $BF = \frac{1}{2} CF = \frac{1}{2}(10) = 5$cm

$\angle ABF = 90°$

so $AF = \sqrt{5^2 + 13^2} = \sqrt{194} \approx 13.9$cm

$\angle AFB = \tan^{-1}\left(\frac{13}{5}\right) \approx 69.0°$

13 From the foot of a building I have to look upwards at an angle of 22° to sight the top of a tree. From the top of the building, 150 metres above ground level, I have to look down at an angle of 50° below the horizontal to sight the tree top.

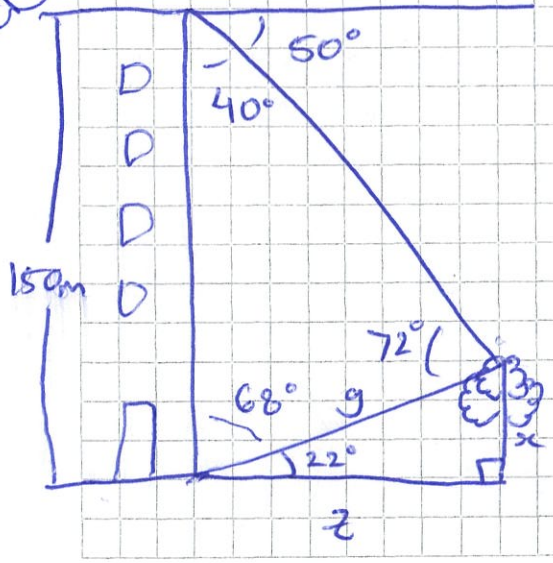
a) How high is the tree?

b) How far from the building is this tree?

the worded nature of this question makes it harder, drawing a good diagram helps.

1. Calc angle (72°)
2. Find y
3. Find x (height of tree).

— once you've done a), this is just SOH CAH TOA



$$\frac{\sin(40°)}{y} = \frac{\sin(72°)}{150}$$

$$y = \frac{150 \times \sin(40)}{\sin(72)} \approx 101.38\text{ cm}$$

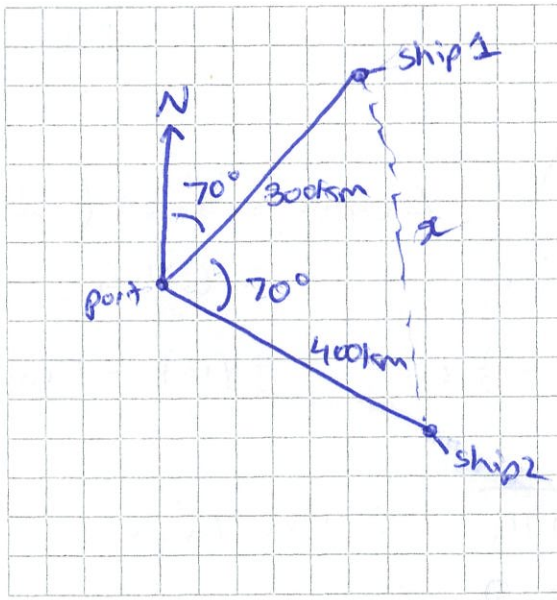
$$\sin(22°) = \frac{x}{y}$$

a) $x = 101.38 \times \sin(22)$

≈ 38.0 cm

b) $z = \sqrt{101.38^2 - 37.98^2} \approx 94.0$m

14 A ship is 300 km from port on a bearing of 070°T. A second ship is 400 km from the same port and on a bearing of 140°T. How far apart, correct to the nearest kilometre, are the two ships?



$$x = \sqrt{300^2 + 400^2 - 2 \times 300 \times 400 \times \cos(70)}$$

≈ 410km.

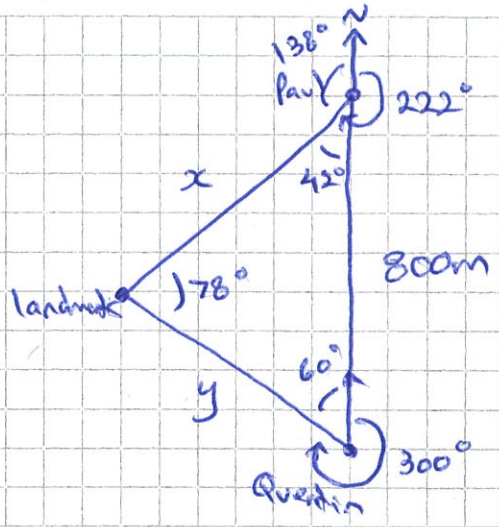
Bearings to interpret the worded question & draw the diagram then it's just the cosine rule once you've found the angle

15 Two hikers, Paul and Quentin, are both looking at a distant landmark. From Paul the bearing of the landmark is 222°T, and from Quentin the bearing of the landmark is 300°T. If Quentin is standing 800 m due South of Paul, find, correct to the nearest metre:

this is the tricky part of this question. Very challenging!

a) the distance from Paul to the landmark

b) the distance from Quentin to the landmark.



a) $\frac{\sin(60)}{x} = \frac{\sin(78)}{800}$

$x = \frac{800 \times \sin(60)}{\sin(78)} \approx 708\text{m}$

b) $\frac{\sin 42}{y} = \frac{\sin(78)}{800}$

$y = \frac{800 \times \sin(42)}{\sin(78)}$

$\approx 547\text{ m.}$